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A pseudo-fluid representation of vertical liquid–coarse solids flow

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Abstract: The pseudo–fluid concept has been applied for the prediction of the pressure gradient and voidage in vertical liquid-coarse solids flow. Treating the flowing mixture as a single homogenous fluid, the correlation for the friction coefficient of the suspension–wall was developed, as was the correlation between the true voidage and the apparent (volumetric) voidage in the transport tube. Experiments were performed using water and spherical glass particles 1.20, 1.94 and 2.98 mm in diameter in a transport tube of 24 mm in diameter. The loading ratio (G_p/G_f) was varied between 0.05 and 1.05 and the fluid superficial velocity was between 0.4 U_t and 4.95 U_t where U_t represents the single particle terminal velocity. The voidage ranged from 0.648 to 0.951 for these ratios. Experimental data for the pressure gradient and voidage from the literature agree well with the proposed correlations.

Keywords: liquid–solids flow, pseudo–fluid representation.

INTRODUCTION

The understanding vertical two-phase liquid–solids flow and related phenomena is of general importance in chemical, biochemical and mechanical processes. In the design of equipment where vertical liquid–solids flow occurs, it is necessary to be able to predict the relationship between the phase velocities, voidage and pressure drop. Some important examples of such equipment are liquid–solids circulating fluidized beds,[1,2] liquid phase draft tube spouted and spout-fluid beds[3,4] and high efficiency heat exchangers.[5] In addition, both vertical and horizontal hydraulic transport of solids suspended in water is well recognized and practiced in the field of mining and mineral processing.[6,7]

In a previous study,[8] a complex one-dimensional steady state model for vertical non-accelerating liquid–solids flow of coarse spherical particles was formulated and verified. The theoretical bases of the model were the continuity and momentum equations for the fluid and particle phase of Nakamura and Capes[9] and the authors' variational

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model for calculating the fluid–particle interphase drag coefficient.[10] Model predictions of the fluid–particle interphase drag coefficient, voidages and pressure gradient in the transport line were in very good agreement with experimental data.

The aim of the present investigation was to check out the application of the pseudo-fluid concept for predicting the pressure gradient and voidage in vertical liquid-coarse solids flow. A two-phase system (suspension) was considered as a single continuous fluid characterized by an apparent density and viscosity. The pseudo-fluid concept has been the basis of several contributions.[11] Recently, Di Felice[12,13] showed that this approach is a useful tool for predicting the settling velocity of foreign particle in a suspension and for predicting phase hold-ups in three phase fluidization.

EXPERIMENTAL

Hydraulic transport experiments were conducted with glass spheres 1.20, 1.94 and 2.98 mm in diameter in water using the apparatus shown in Fig. 1. The transport line (f) was 24 mm in diameter and 1.8 m long. Two pneumatically operated traps (o) detailed in Fig. 1, close off a one meter calibrated section (o-o) trapping particles that settle in the water. This section was calibrated so that the settled height of the particles gives the mass of the particles trapped. It is located far enough above the inlet to the transport line for the flow here to be non-accelerating. The pressure gradient was measured by a water manometer (n), the taps of which were 0.8 m apart. The separation distance between the inlet nozzle and the transport tube inlet (L, Fig. 1) was 20 mm.

The water is introduced at the bottom of the column through the nozzle (a) 20 mm in diameter and through the annular section (b). The water and particle flowrates were measured using a specially designed box (i), which allows all of the flow (fluid and particles) to be collected, separated and weighed. Normally, the particles recirculate and the suspension overflows at (g), while the water overflows at (h). When the fluid and particle flowrates are to be measured, the box (i) is moved to the left to collect the entire flow for a short period of time (10 s to 1 min). The water is then separated from the particles. The particles are dried and weighed and the volume of water is recorded. In each run the fluid and particle mass flowrates, voidage and pressure gradient were measured. The fluid and particle phase velocities were calculated using continuity equations:

$$u = \frac{G_f}{\varepsilon \rho_f A_t} \tag{1}$$

$$v = \frac{G_p}{(1-\varepsilon)\rho_p A_t} \tag{2}$$

A total of 152 data points were collected. The characteristics of the particles, as well as the range of experimental conditions are summarized in Table I.

TABLE 1. Particle characteristics and range of experimental conditions

d_p/mm	1.20	1.94	2.98
ρ_p/(kg/m^3)	2641	2507	2509
U_t/(m/s)[14]	0.188	0.288	0.370
U/U_t	0.74 – 4.94	0.59 – 4.95	0.40 – 3.90
G_p/G_f	0.05 – 1.05	0.18 – 0.98	0.19 – 063
ε	0.662 – 0.951	0.648 – 0.924	0.659 – 0.911

RESULTS AND DISCUSSION

Flowing suspension–wall friction

The one-dimensional suspension momentum equation outside the acceleration zone of the transport tube is[15]

$$-\frac{dP}{dz} = (\rho_p - \rho_f)\, g\, (1-\varepsilon) + F_w \tag{3}$$

where F_w represents the pressure gradient due to suspension-wall friction.

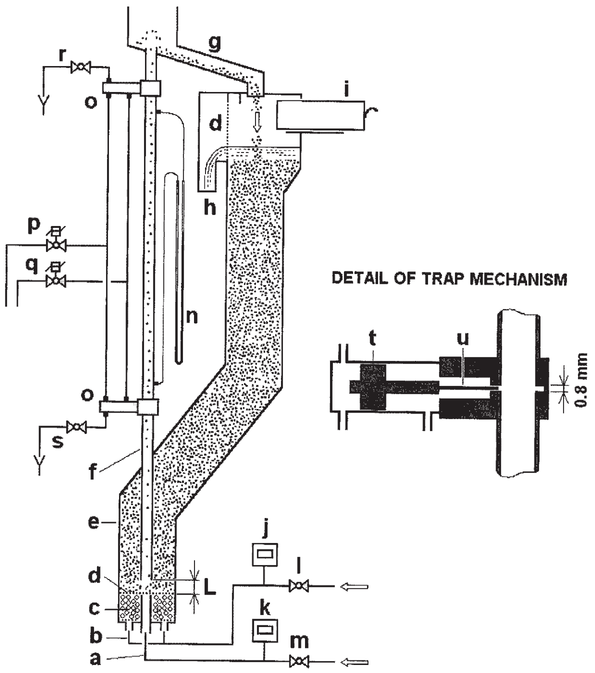

Fig. 1. Schematic diagram of the experimental system (a–nozzle inlet line, b–annulus inlet line, c–distributor, d–screen, e–column, 140 × 140 mm in cross-section, f–draft tube, D_t=24 mm, length 2000 mm, g–particle and water overflow, h–water overflow, i–box for water and particle flowrate measurements, j,k–flowmeters, l, m–valves, n–manometer, o–trap, p–electromagnetic valve for closing the trap, q–electromagnetic valve for opening the trap, r, s–on–off valves, t–piston, u–flowcuttingplane).

The individual momentum balances for the fluid and particle phases (Nakamura and Capes)[9]

$$\varepsilon \left[-\frac{dP}{dz} \right] = \beta (u-v)^2 + F_f \tag{4}$$

$$(1-\varepsilon) \left[-\frac{dP}{dz} \right] = -\beta\, (u-v)^2 + (\rho_p - \rho_f)\, g\, (1-\varepsilon) + F_p \tag{5}$$

where $\beta(u-v)^2$ is the hydrodynamic drag force per unit volume of suspension. F_f and F_p are pressure losses due to fluid–wall and particle-wall friction written in terms of friction factors f_f and f_p.

$$F_f = 2 f_f \rho_f U^2 / D_t \tag{6}$$

$$F_p = 2f_p \rho_p (1-\varepsilon) v^2 / D_t \qquad (7)$$

As seen before, the individual momentum balances for the fluid and particle phases require that the overal friction of the flowing suspension with the wall has an additive character

$$F_w = F_f + F_p \qquad (8)$$

The introduction of separate contributions F_f and F_p (with $F_w = F_f + F_p$) is essentialy a convention, since only the quantity F_w can be determined experimentally if $-dP/dz$ and ε are measured. Using experimental data for $-dP/dz$, U and ε, collected in a previous study,[8] the experimental values of F_w were detgermined from Eq. (3)

$$F_w = -\frac{dP}{dz} - (\rho_p - \rho_f)\, g\, (1-\varepsilon) \qquad (9)$$

Fig. 2. shows the variation of $F_e/(-dP/dz)$ and $F_w/(-dP/dz)$ with superficial fluid velocity, particles having $d_p = 2.98$ mm, where

$$F_e = (\rho_p - \rho_f)\, g\, (1-\varepsilon) \qquad (10)$$

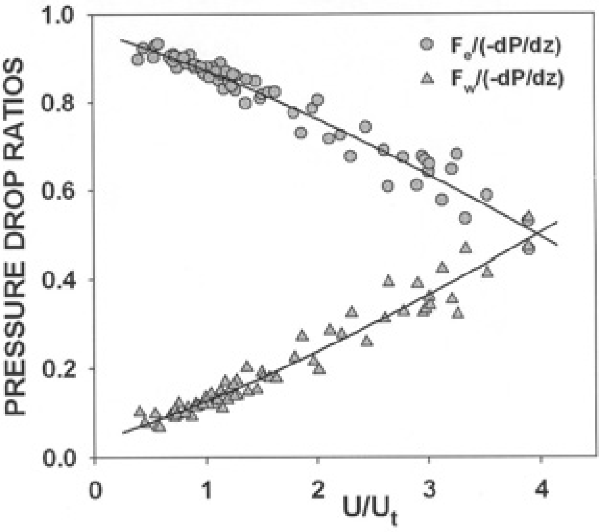

Fig. 2. Variation of $F_e/(-dP/dz)$ and $F_w/(-dP/dz)$ with superficial fluid velocity, d_p=2.98 mm.

At low relative velocities where the solids fraction is high, the major portion of the dynamic pressure drop is due to the static head of the submerged particles (F_e). With increasing liquid velocity, the fluid–wall fraction contribution increases significantly and can be as high as about 50 % of the total at $U/U_t \approx 4$.

Treating a flowing suspension as a pseudofluid with an average density

$$\rho_m = \varepsilon \rho_f + (1-\varepsilon)\rho_p \qquad (11)$$

a modified suspension-wall friction coefficient can be defined in analogy with Eq. (6):

$$f_{\mathrm{w}} = \frac{F_{\mathrm{w}} D_{\mathrm{t}}}{2 \rho_{\mathrm{m}} U_{\mathrm{m}}^{2}} \tag{12}$$

where the average suspension superfical velocity is defined as the total volumetric flowrate per unit cross-sectional area of the transport tube

$$U_{\mathrm{m}} = \frac{V_{\mathrm{j}} + V_{\mathrm{p}}}{A_{\mathrm{t}}} = \frac{G_{\mathrm{j}}}{\rho_{\mathrm{f}} A_{\mathrm{t}}} + \frac{G_{\mathrm{p}}}{\rho_{\mathrm{p}} A_{\mathrm{t}}} = U + c_{\mathrm{s}} \tag{13}$$

The modified Reynolds number for the flowing suspension is

$$Re_{\mathrm{m}} = \frac{D_{\mathrm{m}} \rho_{\mathrm{m}} U_{\mathrm{m}}}{\mu_{\mathrm{m}}} \tag{14}$$

where the effective flowing suspension viscosity is given by Barnea and Mizrahi[16]

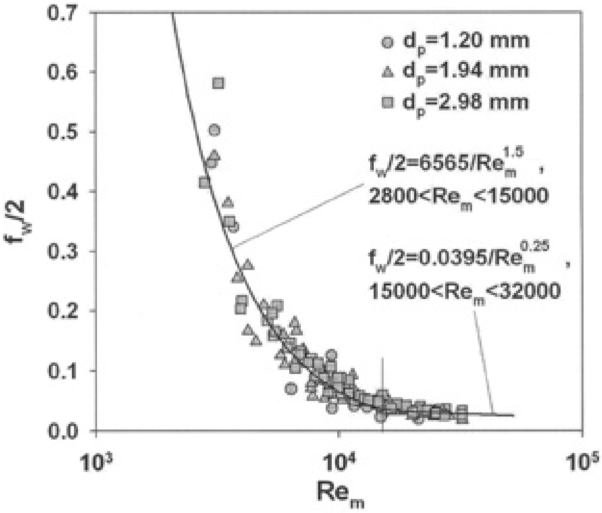

Fig. 3. Correlation of data for the coefficient of the suspension–wall friction.

$$\mu_{\mathrm{m}} = \mu \exp\left(\frac{5(1-\varepsilon)}{3\varepsilon}\right) \tag{15}$$

Fig. 3. gives experimental values of $f_{\mathrm{v}}/2$ as a function the Reynolds number of the suspension. These data are correlated by the equations:

$$f_{\mathrm{w}}/2 = 6565/Re_{\mathrm{m}}^{1.50}, \quad 2800 < Re_{\mathrm{m}} < 15000 \tag{16}$$

and

$$f_{\mathrm{w}}/2 = 0.0395/Re_{\mathrm{m}}^{0.25}, \quad 15000 < Re_{\mathrm{m}} < 32000 \tag{17}$$

The form of the correlation for $Re_{\mathrm{m}} > 15000$ is the same as the well-known Blasius equation for single-phase flow[17] in smooth tubes

$$f_f/2 = 0.0395/Re^{0.25} \tag{18}$$

where *Re* represent the pipe Reynolds number for single-phase flow. Eq. (18) is valid for transient and turbulent flow regimes, *i.e.,* for 2300 < *Re* < 10.[5] The mean absolute deviation between the experimental data and Eq. (16) is 15.5 %, while for Eq. (17), the mean absolute deviation is 13.8 %.

Volumetric and true voidage

Voidage in the transport tube (ε) is defined as the ratio (volume of the fluid)/(volume of the fluid + volume of the particles) in the control volume of the transport tube. Using the volumetric flowrate of the fluid and of the particles, the apparent (volumetric) voidage can be defined:

$$\varepsilon_V = \frac{V_f}{V_f + V_p} \tag{19}$$

Since $V_f = G_f/\rho_f$ and $V_p = G_p/\rho_p$, by combining Eq. (19) with Eqs. (1) and (2) the relationship between ε_V and ε is

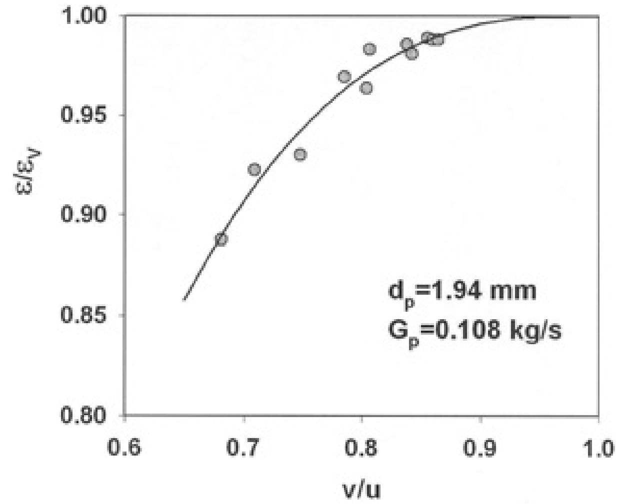

Fig. 4. Relationship $\varepsilon/\varepsilon_V$ *vs.* v/u for suspension flow, d_p = 1.94 mm, G_p = 0.108 kg/s.

$$\frac{\varepsilon}{\varepsilon_V} = \varepsilon + \frac{v}{u}(1-\varepsilon) \tag{20}$$

The true voidage is always less than the volumetric voidage since the particle velocity is (due to the slip) less than the mean interstitial fluid velocity. The ratio $\varepsilon/\varepsilon_V$ approaches 1 when $v/v \rightarrow 1$, as illustrated in Fig. 4. For a quick estimate of the true voidage, a correlation for the ratio $\varepsilon/\varepsilon_V$ would be useful. Fig. 5. shows that the correlation from can be

$$\frac{\varepsilon}{\varepsilon_V} = \frac{9.78(U/U_t)}{1+9.63(U/U_t)} \tag{21}$$

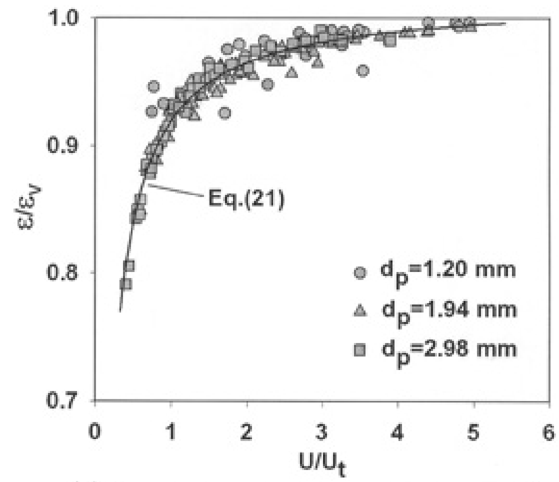

Fig. 5. Correlation of the dfata for $\varepsilon/\varepsilon_V$ in suspension flow.

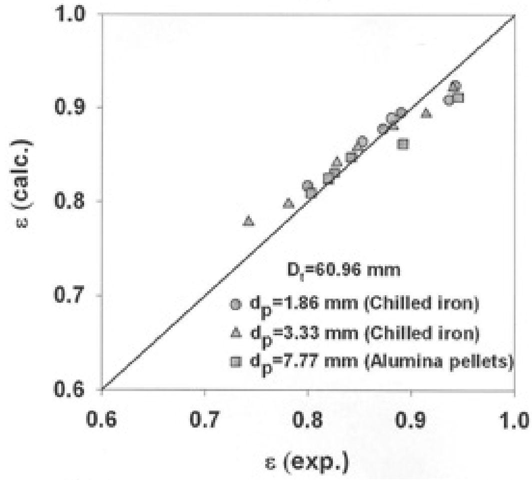

Fig. 6. Comparison of the measured and calculated values of the voidage in the transport tube. Data from Kopko *et al.*[18]

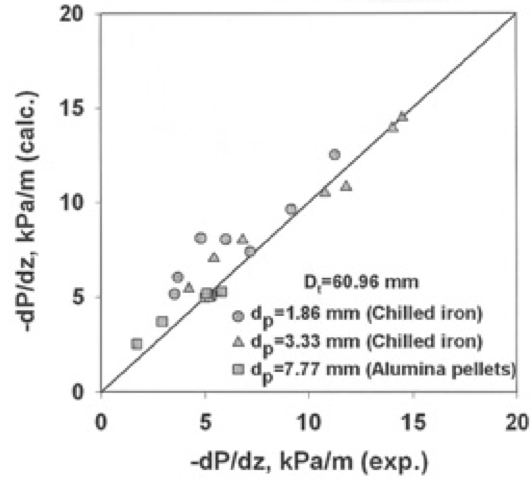

Fig. 7. Comparison of the measured and calculated values of $-\,dP/dz$ in the transport tube. Data from Kopko *et al.*[18]

The mean absolute deviation between the experimental data and Eq. (21) is 4.5 %.

The only consistent set of the experimental data from the literature which could be used to check the applicability of Eqs. (21) and (3), (16) and (17) are the data of Kopko *et al.*[18] These authors transported chilled iron and alumina pellets with water in a transport tube of D_t = 60.96 mm. Fig. 6. gives the comparison between the experimental and the calculated values of the voidage, while Fig. 7. gives the comparison between the experimental and the calculated values of the pressure gradient in the transport tube. The agreement between the calculated and the measured values is quite good, the mean absolute deviation in the prediction of the voidages is 1.63 %, while the mean absolute deviation in the prediction of the preessure gradients is 19.5 %.

CONCLUSIONS

The pseudo-fluid concept was applied to the prediction of the pressure gradient and the voidage in vertical liquid-coarse solids flow. Treating the flowing mixture as a single homogeneous fluid, the correlation for the coefficient of the suspension–wall friction was developed, as was the correlation between the true voidage and the apparent (volumetric) voidage in the transport tube. Experiments were performed using water and spherical glass particles 1.20, 1.94 and 2.98 mm in diameter and a transport tube of 24 mm in diameter. The experimental data of Kopko *et al.*[18] agree quite well with the proposed correlations.

NOMENCLATURE

A_t	Cross-sectional area of the transport tube (m^2)
c_s	Particle superficial velocity in the transport tube, = $G_p/\rho_p A_t$(m/s)
d_p	Particle diameter (m)
D_t	Diameter of the transport tube (m)
f_f	Coefficient of fluid-wall friction
f_p	Coeficient of particle-wall friction
f_w	Coeficient of suspension-wall friction
F_e	Pressure gradient due to the effective weight of the particles (*P*a/m)
F_f	Pressure gradient due to the fluid-wall friction (*P*a/m)
F_p	Pressure gradient due to the particle-wall friction (*P*a/m)
F_w	Pressure gradient due to the suspension-wall friction (*P*a/m)
g	Gravitational acceleration (m/s^2)
G_f	Fluid mass flowrate in the transport tube, = $\rho_f A_t U$ (kg/s)
G_p	Particle mass flowrate in the transport tube, = $\rho_p A_t v(1–\varepsilon) = \rho_p A_t c_s$ (kg/s)
L	Separation distance between the inlet nozzle and the transport tube inlet (see Fig. 1) (m)
P	Dynamic pressure (*P*a)
Re	Pipe Reynolds number, = $D_t \rho_f U/\mu$
Re_m	Modified suspension Reynolds number, = $D_t \rho_m U_m/\mu_m$
u	Mean interstitial fluid velocity in the transport tube, = U/ε (m/s)
U	Superficial fluid velocity in the transport tube (m/s)
U_m	Superficial suspension velocity, = $U + c_s$(m/s)

U_t	Particle terminal velocity in an infinite medium (m/s)
v	Radially averaged particle velocity in the transport tube (m/s)
V_f	Water volumetric flowrate through the transport tube (m^3/s)
V_p	Particle volumetric flowrate through the transport tube (m^3/s)
z	Vertical coordinate (*m*)

Greek letters

β	Fluid-particle interphase drag coefficient (kg/m^4)
ε	Radially averaged voidage in the transport tube
ε_v	Volumetric voidage in the transport tube, defined by Eq. (19)
μ	Viscosity of the fluid (Ns/m^2)
μ_m	Viscosity of the fluid-particle suspension (Ns/m^2)
ρ_f	Fluid density (kg/dm^3)
ρ_p	Particle density (kg/m^3)
ρ_m	Average suspension density = $\varepsilon\rho_f(1-\varepsilon)\rho_p$ (kg/m^3)

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ИЗВОД

ВЕРТИКАЛНО КРЕТАЊЕ ТЕЧНОСТИ И ЧВРСТИХ ЧЕСТИЦА ТРЕТИРАНО КАО ПСЕУДО–ФЛУИД

РАДМИЛА ГАРИЋ-ГРУЛОВИЋ,[1] ЖЕЉКО ГРБАВЧИЋ[2] и ЗОРАНА АРСЕНИЈЕВИЋ[1]

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За предвиђање градијента притиска и порозности при вертикалном двофазном току течности и крупних чврстих честица примењен је концепт псеудофлуида. Третирајући покретну смешу течности и честица као хомогени флуид, одређена је зависност за коефицијент трења суспензија–зид транспортне цеви, као и зависност између стварне и запреминске (волуметријске) порозности у транспортној цеви. Експериментална испитивања су изведена у транспортној цеви пречника 24 mm, са сферичним стакленим честицама пречника 1.20, 1.94 и 2.98 mm, при чему је као транспортни медијум коришћена вода. Оптерећење тока (G_p/G_f) варирало је између 0.05 и 1.05, док се површинска брзина флуида кретала од 0.4.U_t до 4.95.U_t, где је U_t брзина одношења усамљене честице. За наведене обиме оптерећења тока и брзине флуида порозност у систему се кретала од 0.648 до 0.951. Експериментални подаци из литературе за градијент притиска и порозност у доброј су сагласности са предложеним корелацијама.

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