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Resonant Frequency and Phase Noise of Nanoelectromechanical Oscillators Based on Two-Dimensional Crystal Resonators

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Abstract

This study presents a theoretical analysis of one of the most promising nanoelectromechanical (NEMS) components – a NEMS oscillator. The analyzed oscillator contains a stretched circular plate, fabricated of two-dimensional crystals (graphene, bBN, MoS_2 etc.), as a resonator. The calculation of resonant frequency based on the classical continuum theory of plates and membranes is presented, and then the phase noise theory of the oscillators using a circular plate as a frequency determining element. We assume that thermal and 1/f noise are present in the oscillator circuit. A satisfactory agreement is obtained between our calculations and recent experimental literature data for graphene.

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Keywords: Graphene oscillator; NEMS oscillator; two-dimensional crystal resonator; phase noise

1. Introduction

The emergence of atomic thin solid-state structures – two-dimensional (2D) crystals (e.g. graphene, bBN, MoS_2 etc.) enables the development of nanoelectromechanical (NEMS) systems with new and improved functionalities and smaller dimensions [1]. There is a growing interest in the development of NEMS oscillators with a frequency determining element based on the mentioned 2D structures; these oscillators have various applications in electronics and highly sensitive sensors [2,3].

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In this study, we present a theoretical analysis of a NEMS oscillator with a stretched circular plate as a resonator. The plate is fabricated of 2D crystals. After the calculation of resonant frequency based on the classical continuum theory of plates and membranes, the phase noise theory of the oscillators using a circular plate as a frequency determining element will be given, considering thermal noise and 1/f noise in the oscillator circuit. The calculation results will be compared with recent experimental literature data on phase noise for graphene oscillators.

2. Theoretical considerations

The resonant frequency of a stretched circular plate of a radius a (Fig. 1) may be obtained using classical continuum mechanics assuming that deflections occur in the region of elastic strain [4,5]. The expression for the lowest mode (mode (0,0)) resonant frequency, for the case of clamped structure, is

$$f_{r,\varepsilon} = \frac{(k_2a)_\varepsilon}{2\pi a}\sqrt{\frac{(k_2a)_\varepsilon^2 D}{\rho_{2D}a^2} + \varepsilon\frac{Eh}{\rho_{2D}}}\ ,\ (k_2a)_\varepsilon^2 = 5.7832 + 4.4318\cdot e^{-0.1148(\varepsilon a^2 Eh/D)^{0.4863}} \quad (1)$$

where $D=Eh^3/(12(1-v^2))$ is the bending stiffness, E is Young modulus in [N/m^2], ρ_{2D} is density in [kg/m^2], v is the Poisson ratio, h is the thickness of the resonant structure, and ε is the strain. The effect of strain on the resonant frequency can be expressed through the ratio

$$\frac{f_{r,\varepsilon}}{f_{r,0}} = \frac{(k_2a)_\varepsilon^2}{10.215}\sqrt{1+\varepsilon\frac{12(1-v^2)}{(k_2a)_\varepsilon^2}\left(\frac{a}{h}\right)^2} \quad (2)$$

$f_{r,0}$ being the resonant frequency of the same structure in the absence of strain, and the eigenvalue of clamped non-streched plate $(k_2a)_0^2$ equals 10.215, both obtained from Eqs. (1) for ε=0.

The oscillator phase noise is an important parameter, since it determines the oscillator short-therm frequency stability, and therefore influences the signal transmission quality in communication systems or the sensor limiting performance (such as the minimal detectable signal). The oscillator phase noise originates from phase fluctuations caused by noise generation mechanisms both in the oscillator's electronic circuit and in the resonator. The phase noise of the oscillator expressed in [dBc/Hz] is

$$L(f) = 10\log(S_\theta(f)) \quad (3)$$

where f is the offset frequency from f_r, and $S_\theta(f)$ is the power spectral density of the oscillator signal phase fluctuations, obtained by using the Wiener-Khinchine theorem in the form [6,7]

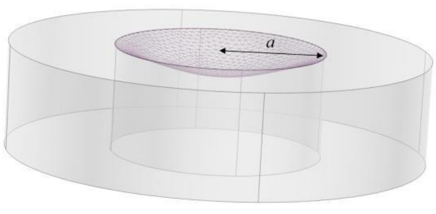

Fig. 1. Illustration of the stretched circular plate made of 2D crystal and used as a frequency determining element of a NEMS oscillator. The shown deflection corresponds to the lowest resonant mode.

$$S_{\theta}(f) = \int_{-\infty}^{+\infty} e^{-\sigma^2(|\tau|)/2} e^{-j2\pi f\tau} d\tau \tag{4}$$

By assuming that thermal and 1/f noise are present in the oscillator circuit [6,7] the total variance of the phase increments equals the sum of the corresponding variances

$$\sigma^2(\tau) = 2D_{\varphi}\tau + K\tau^2 \tag{5}$$

Here, D_{φ} is the diffusion constant of the Brownian motion of phase, which is related to the oscillator thermal noise, and K is related to parameters of the 1/f noise in an oscillator circuit.

3. Numerical results and discussion

Fig. 2a shows the graphene circular plate resonant frequency dependence on strain, obtained using Eqs. (1), for the plates of different radii (a=1 μm or a=2 μm), consisting of different numbers of atomic layers, n (n=1, 3 or 7). Two regions of strain for which either bending (the region where f_0 weakly depends on strain) or tension (where the dependence $f_0 \sim \varepsilon^{1/2}$ is approximately valid) dominate can be observed. In the latter region, it is possible to achieve the tunability of the oscillator resonant frequency [3], and also the extremely high resonant frequency values. Namely, the high stiffness (E=1 TPa) and low mass of graphene lead to high natural resonant frequencies $f_{r,0}$, which can be further increased by the application of strain. For example, by applying the strain ε=1%, the resonant frequency of a single-layer graphene structure with a radius of 1 μm increases from about 3.5 MHz to almost 1 GHz (Fig. 2a). In resonant structures fabricated from 2D materials, the values of ε at which it is possible to externally adjust resonant frequency over a wide range are considerably lower than those in thicker (3D) structures. This is clear from Eq. (2), which shows that the minimum value of ε, at which $f_0 \sim \varepsilon^{1/2}$, is proportional to $(h/a)^2$. The lower the ratio of the thickness and the radius of a resonant structure, the greater the change of resonant frequency (the greater the ratio $f_{r,\varepsilon}/f_{r,0}$) with the application of the same strain.

Fig. 2b shows the phase noise of the nanomechanical oscillator based on 2D crystal resonant structure (one layer thick graphene circular plate of diameter 2 μm). The solid line is obtained by numerical integration, based on Eqs. (3)-(5). The parameters D_{φ} and K are determined by fitting the experimental data given in Ref. [3]. The experimental data from the same Ref. are also shown in the diagram (discrete lines). The agreement between data is satisfactory, showing that both thermal and 1/f noise have a significant influence on the phase noise of these oscillators.

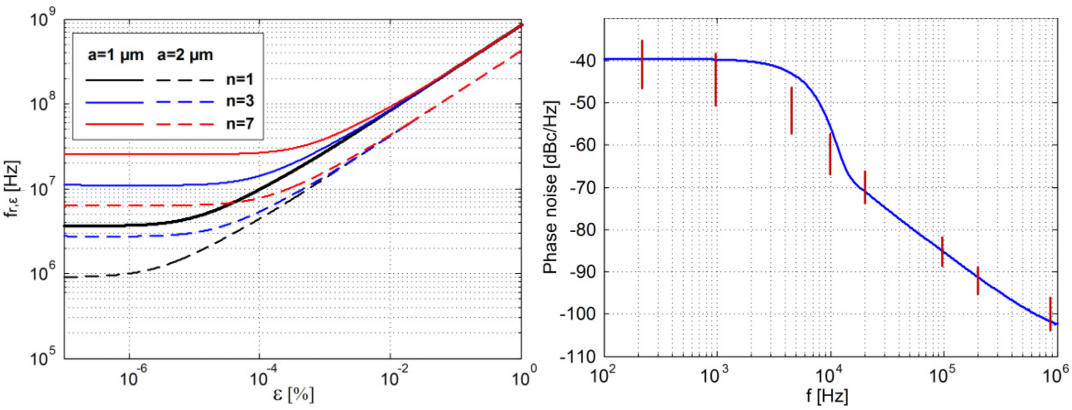

Fig. 2. (a) The graphene circular plate resonant frequency dependence on strain, for two different plate radii: a=1 μm (solid lines) or a=2 μm (dashed lines), and three different thicknesses, given by the number of atomic layers, n: n=1 (black), n=3 (blue) or n=7 (red); (b) The oscillator phase noise obtained by numerical integration (blue solid line). The experimental data (Ref. [3]) are shown in red (discrete lines).

4. Conclusion

A theoretical analysis of nanoelectromechanical oscillators using two-dimensional crystal resonators as a frequency determining element has been presented. The effect of strain on the resonant frequency of a graphene stretched circular plate has been analyzed. Due to the exceptional mechanical properties of graphene and an extremely small thickness of the resonant structure, extremely high resonant frequencies can be obtained and they can be further increased by the application of strain. The lower the ratio of the thickness and the radius of a resonant structure, the greater the relative change in resonant frequency with the application of a certain amount of strain, as compared to the resonant frequency in the absence of strain. The calculations presented in the paper have shown that the application of no more than one-percent strain the resonant frequency of a single-layer graphene structure with a radius of 1 μm increases from about 3.5 MHz to almost 1 GHz.

In this study, the presented theory of the phase noise is for the first time applied in nanomechanical oscillators based on 2D crystals. The satisfactory agreement between our theoretical results and the experimental data from the literature shows that both thermal and 1/f noise have a significant influence on the phase noise of these oscillators.

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