HEAT TRANSFER IN THE VERTICAL TWO-PHASE FLOW OF COARSE PARTICLES

by

Radmila V. GARIĆ-GRULOVIĆ, Željko B. GRBAVČIĆ, Zorana Lj. ARSENIJEVIĆ, and Stevan Dj. JOVANOVIĆ

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Wall-to-bed heat transfer in hydraulic transport of spherical glass particles 1.20, 1.94, and 2.98 mm in diameter and in single-phase flow regime was studied. Experiments were performed by transporting spherical glass particles with water in a 25.4 mm I. D. copper tube equipped with a steam jacket.

In the runs with particles not present tube Reynolds number varied between 2280 and 21300, while in hydraulic transport runs tube Reynolds number varied between 3300 and 20150. The loading ratio (G_p/G_f) *was between 0.07 and 0.328, and the fluid superficial velocity was between* $0.29 \cdot U_t$ *and* $2.86 \cdot U_t$, *where* U_t *represents single particle terminal velocity.*

The data for the heat transfer factor (j_H) *in single phase flow are correlated using general form* $j_H = f(Re)$. *The data for wall-to-bed heat transfer in the hydraulic transport of particles shows that analogy between heat and momentum transfer exists. The data were correlated by treating the flowing fluid-particle mixture as a pseudofluid, by introducing a modified mixture-wall friction coefficient* (f_w) *and a modified mixture Reynolds number* (Re_m).

Introduction

The understanding of vertical two-phase liquid-solids flow and related phenomena is generally important in chemical and biochemical processes. Hydraulic transport of solids suspended in water, both vertical and horizontal, is well recognized and practiced in the field of mining and mineral processing [1].

Our interest in the subject is primarily associated with the development of new liquid-solids spouted and spout-fluid beds with draft tubes because of their desirable characteristics as chemical and biochemical reactors. The spouted or spout-fluid bed with a draft tube have complex hydrodynamics and consist essentially of two parts: (1) the draft tube where fluid and particles move upwards as dense or dilute phase mixture, and (2) the annulus surrounding the draft tube where particles move slowly downward as a loosely packed bed.

In contrast to the large number of investigations of wall-to-bed heat transfer in gas-fluidized and liquid-fluidized beds [2–4], or in vertical gas-solids flow [5], there is no data, in the available literature, on the wall-to-bed heat transfer for the hydraulic transport of coarse particles.

For single-phase flow, there are a large number of wall-to-fluid heat transfer investigations. From the phenomenological point of view, the most important result is well known Chilton-Colburn analogy [6]. For turbulent flow conditions, these authors found

$$j_H = j_D = f_f/2 \tag{1}$$

where j_H is heat transfer factor, j_D is mass transfer factor and f_f is fluid-wall friction coefficient.

The aim of the present investigation was to study effect of particles on wall-to-flowing mixture heat transfer and attempt to relate wall-to-flowing mixture heat transfer with the friction between the heating wall and flowing mixture.

Experimental

An outline of the experimental apparatus is shown in Fig. 1. The hydraulic and heat transfer experiments were conducted with glass spheres 1.20, 1.94 and 2.98 mm in diameter in water. The transport line (c) was a copper tube 27.4/25.4 mm in diameter and 1360 mm long equipped with a 700 mm long steam jacket. The tube was mounted in a modified spouted bed in order to obtain non-fluctuating controlled flow of particles. The heating section (d) was located far enough (320 mm) above the inlet to the transport line for the flow there to be non-accelerating. At the bottom of the bed the water is introduced through a nozzle (a) which is 20 mm in diameter. The separation distance between the bed bottom and the transport tube inlet (L, Fig. 1) was 20 mm.

The fluid and particle flowrates are measured using a specially designed box (g), which allows all of the flow (fluid and particles) to be collected, separated and weighed. Normally, the particles recirculate and the fluid overflows at (e). When the fluid and particle flowrates are to be measured the box (g) is moved to the left to collect the entire flow for a short period of time (10 s to 1 min). The water is then separated from the particles. The particles are weighed dry and the volume of water recorded. The pressure gradient was measured using piezometers.

Saturated steam at atmospheric pressure was supplied to the steam jacket. The inlet water temperature was maintained constant during each experimental run. It ranged from 14 to 16 °C. Temperatures were measured with Ni-Cr thermocouples. The wall temperature was measured at two points (inlet and exit) on a such way that junction point was filled with tin at about 0.2 mm from the inside tube wall, as shown schematically on Fig. 1 (detail "A"). The exit temperature of the fluid-particle mixture was measured by a thermocouple located in the tube axis. It was assumed that at the inlet and at the exit of the heating zone, the particles and fluid have the same temperature. Note that in a

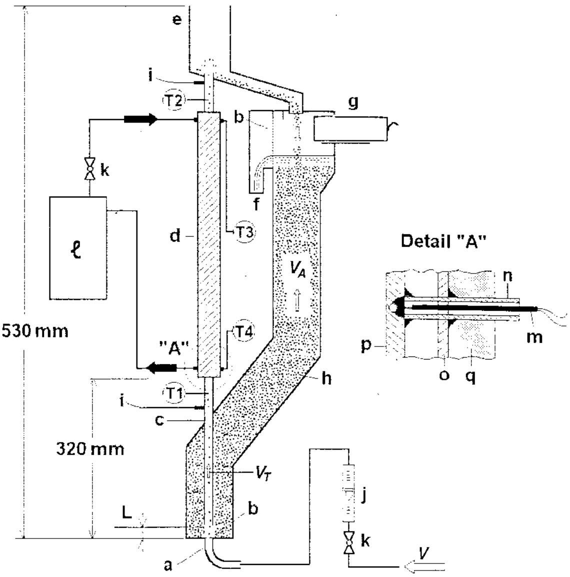

Figure 1. Schematic diagram of experimental system
(a) inlet nozzle, 20 mm i. d., (b) screen, (c) transport tube, 25.4 mm i. d., (d) heating section, 700 mm in length, (e) overflow, (f) water overflow, (g) box, (h) modified sputed bed, 70×70 mm in cross-section, (i) pressure taps, (j) rotameter, (k) valve, (l) steam generator, 30 kW, (m) thermocouple, (n) copper tube 8/6 mm, (o) jacket wall, (p) transport tube wall, (q) thermoisolation

transport system mounted in the modified spouted bed, the inlet flowrate (V, Fig. 1) is divided into the tube flow (V_T) and annular flow (V_A). In our experiments, the ratio V_A/V varies between 0.07 and 0.52, so that heated particles falling into the annulus region have enough time to approach inlet water temperature. The particle residence time in the heating section of the transport tube varied between 0.9 and 60 s. Assuming that the particles and fluid have the same temperature at exit, the heat transfer coefficient is

$$h = \frac{(G_f C_{pf} + G_p C_{pp})(T_1 - T_2)}{(D_t \pi L_H)\Delta T_{lm}} \tag{2}$$

where G_f is fluid flowrate, G_p is particle flowrate, C_{pf} is fluid specific heat, C_{pp} is particle specific heat, D_t is inner tube diameter and L_H is length of the heating zone. The mean logarithmic temperature difference is defined

$$\Delta Tl_{lm} = \frac{(T_3 - T_2) - (T_4 - T_1)}{\ln\dfrac{(T_3 - T_2)}{(T_4 - T_1)}} \tag{3}$$

Particle characteristics as well as the range of experimental conditions are summarized in Table 1. A total of 155 data points were collected in the runs with particles not present ($G_p = 0$). In these runs, the tube Reynolds number varied between 2280 and 21300. A total of 74 data points on heat transfer coefficients were collected during

Table 1. Particle characteristics and range of experimental conditions

d_p [mm]	1.20	1.94	2.98
ρ_p [kg/m^3]	2641	2507	2509
U_t [m/s] [8]	0.1884	0.2878	0.3698
U/U_t	0.43–2.15	0.31–2.86	0.29–2.11
W_p [kg/m^{2}s]	6.5–86.9	0.7–226.2	6.0–239.3
G_p/G_f	0.08–0.237	0.08–0.302	0.07–0.328
ε	0.780–0.895	0.751–0.0884	0.715–0.864

hydraulic transport runs. In these runs, the tube Reynolds number varied between 3300 and 20150. Under these conditions the loading ratio (G_p/G_f) was between 0.07 and 0.328, and the fluid superficial velocity was between $0.29 \cdot U_t$ and $2.86 \cdot U_t$, where U_t represents single particle terminal velocity. For these ratios the voidage ranged from 0.715 to 0.895. Voidage in the transport line was calculated using measured values of particle mass flowrate (G_p), fluid mass flowrate (G_f) and pressure gradient ($-dP/dz$) and the model described in detail in our previous work [7]. Here we have assumed that particle wall friction term is unaffected by the transport tube material and water temperature, so that correlation for particle – wall friction term developed in our previous work [7] was used. This correlation was developed using the same spherical glass particles (1.2, 1.94, and 2.98 mm) and glass transport tube of the same diameter. The only difference is that in previous work cold water (14–16 °C) was used as a transport media.

Results and discussion

Single-phase flow

Figure 2 gives the relationship between heat transfer factor and Reynolds number in single-phase flow conditions. The data are correlated separately for transition and turbulent flow regimes, using Colburn type equations:

$$j_H = 3940 / \mathrm{Re}^{1.50}, \quad 2300 < \mathrm{Re} < 10000 \tag{4}$$

$$j_H = f_f / 2 = 0.0395 / \mathrm{Re}^{0.25}, \quad 10000 < \mathrm{Re} < 21300 \tag{5}$$

The mean deviation of the data from eq. (4) is 21.29% on average. As can be seen, our data for wall-to-fluid heat transfer in turbulent flow regime (Re < 10000) agree

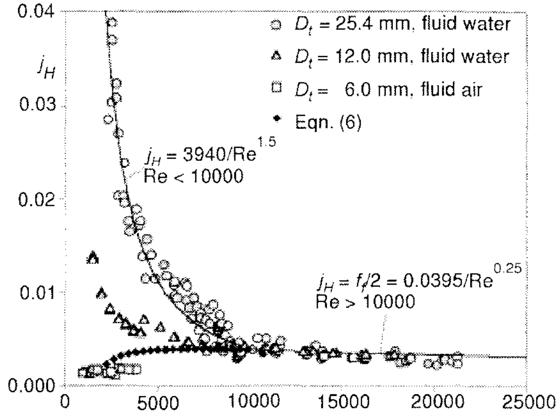

Figure 2. Correlation of experimental data for heat transfer factor (single phase flow)

quite well with the Chilton-Colburn [6] analogy, $j_H = f_f/2$, where f_f is fluid-wall friction factor. The mean deviation of data from eq.(5) is 13.2% on average. A comparison of the data for single phase flow with several literature correlations [9] shows satisfactory agreement in turbulent regime (Re > 10000). However, in transition regime (2300 < Re < < 10000) there is significant difference between available correlations and our data. In the same figure some preliminary data for water heating in tube D_t = 12.0 mm and data for air heating in tube D_t = 6.0 mm are plotted. As seen, there are significant differences among the systems investigated. Gnielinski [10] proposed following correlation for heat transfer in smooth straight tubes:

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re}-1000)\,\mathrm{Pr}}{1-12.7\sqrt{(f/8)}(\mathrm{Pr}^{2/3}-1)}\left[1+\left(\frac{D_t}{L_H}\right)^{2/3}\right] \tag{6}$$

where fluid-wall friction coefficient is given by the following correlation

$$f = [1.82\cdot\log(\mathrm{Re}) - 1.64]^{-2} \tag{7}$$

Equations (6) and (7) represents the majority of the data within 20% and it is recommended in the following range of variables: $0 < D_t/L_H < 1$, $0.6 < \mathrm{Pr} < 2000$ and $2300 < \mathrm{Re} < 10^6$. Eqs. (6 and 7) are also plotted in Fig. 2, in the form $j_H = (\mathrm{Nu/RePr}^{1/3}) = = f(\mathrm{Re})$. Note that fluid-wall friction coefficient given by eq. (7) differs from f_f given by Eq. (1) by a factor 4 ($f_f = f/4$) due to the different definitions. It seems that our results support the view [11] that the heat transfer results in the transition regime are uncertain because of the large number of parameters which determine when and how transition occurs.

Two-phase flow

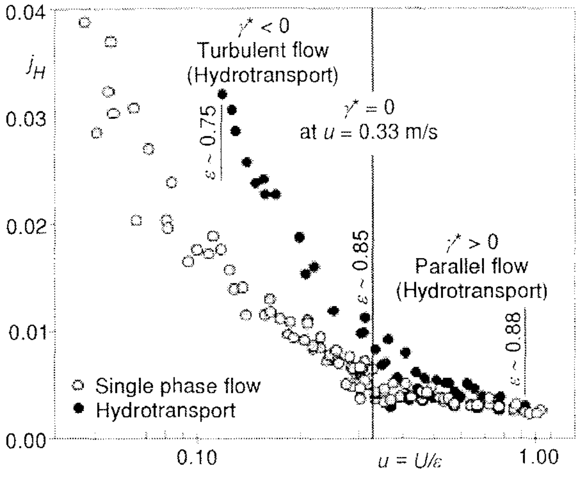

Figure 3. j_H ***vs.*** u **for single phase flow and hydraulic transport (**d_p **= 1.94 mm)**

Figure 3 gives variation of wall-to-bed heat transfer for the hydraulic transport of 1.94 mm diameter particles with water. In the same plot, the data for j_H factor in single phase flow are plotted for the purpose of comparison. As can be seen for high fluid velocities there is no significant difference. The presence of particles improves heat transfer for lower fluid velocities, where the particle concentration in flowing mixture is generally higher and where flow regime is different. As can be seen with decrease in liquid velocity, the heat transfer factor for two phase flow increases relative to the values in single phase flow by a factor of about 2.2.

Flow regime

In the hydraulic transport of coarse particles, we have observed two characteristic flow regimes:

(a) "Turbulent" flow where the particles moves vertically but with some radial movement. This regime is characteristic of the lower fluid and particle velocities and appears very much like the settling of a particle suspension upside down. At the same time, the flowing mixture is very much like a particulately fluidized bed, where the whole "fluidized" mixture flows relative to the tube walls.

(b) "Parallel" flow where the particles move vertically along a parallel streamline. This regime is characteristic of the higher fluid and particle velocities.

A schematic representation of these flow regimes is given in Fig. 4. In our previous work [7] we found that the choking criterion for vertical pneumatic transport lines of Day *et al.* [12] works also as a criterion for regime designation in the hydraulic transport of coarse particles. Day *et al.* [12] studied choking phenomena in vertical two-phase gas–solids flow. Using steady state of one-dimensional mixture momentum equation, with constant fluid properties including frictional effects between the bed and the wall [13]

$$-\frac{dP}{dz} = (\rho_p - \rho_f)g(1-\varepsilon) + F_w + \gamma\frac{d\varepsilon}{dz} \tag{8}$$

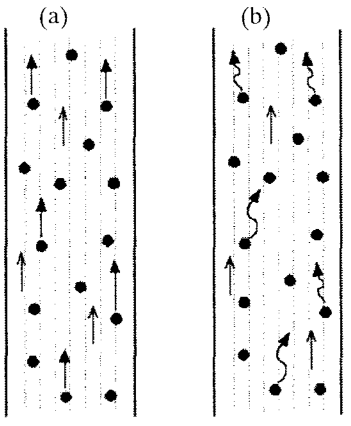

Figure 4. Schematic representation of "parallel" (a) and "turbulent" (b) particle flow regime

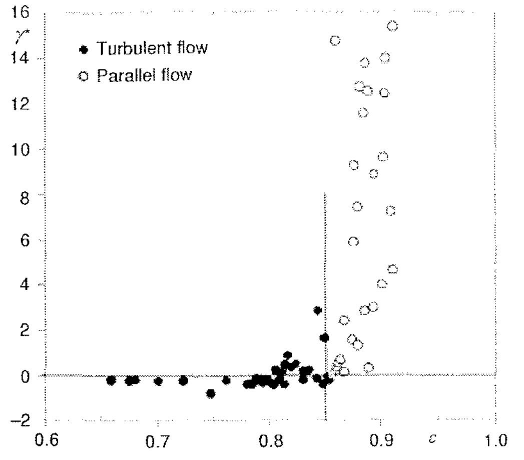

Figure 5. Relationship γ^* *vs.* ε for hydraulic transport, d_p = 1.94 mm

where

$$\gamma = \rho_p v^2 - \rho_f u^2 \tag{9}$$

they proposed $\gamma = 0$ at the inlet to the transport tube as their choking criterion. This criterion together with a semi-theoretical relationship for the slip velocity at the inlet leads to an equation for prediction of the choking velocity which agrees quite well with all available experimental data for vertical gas-particle flow. Our visual observations indicate that $\gamma = 0$ in hydraulic transport corresponds to the transition from "turbulent" to the "parallel" flow. Fig. 5 shows the relationship between dimensionless parameter γ^* and transport line voidage, where

$$\gamma^* = \frac{\rho_p v^2 - \rho_f u^2}{\rho_f U_t^2} \tag{10}$$

On the same plot, the points according to the visual regime identification are designated. As can be seen, the critical voidage for regime transition is about $\varepsilon \approx \approx 0.85$. Since in the "turbulent" flow regime, the frequency of particle collisions with the tube wall is much higher, it reasonable to expect higher heat transfer rate in this flow regime. Fig. 6 shows that this is indeed the case, since heat transfer factors rapidly increase when $\gamma^* < 0$. The same conclusion can be drawn from Fig. 3, since in this figure the transition line is also designated. For d_p = 1.94 mm particles the parametar $\gamma^* = 0$ at fluid interstitial velocity u = 0.33 m/s. "Turbulent" flow regime appears very much like a particulately fluidized bed so that one can expect similar values of j_H factors for "turbulent" hydro-transport and particulate fluidization. Figure 7 shows that this is really the case. Heat transfer factors for same particles fluidized by water or transported by water in the "turbulent" regime are practically identical. Note that the fluidization experiments [14]

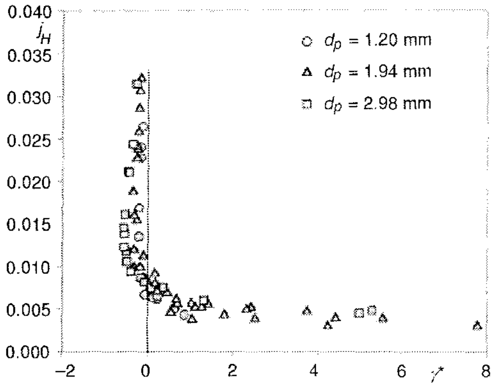

Figure 6. Relationship j_H vs. γ^*

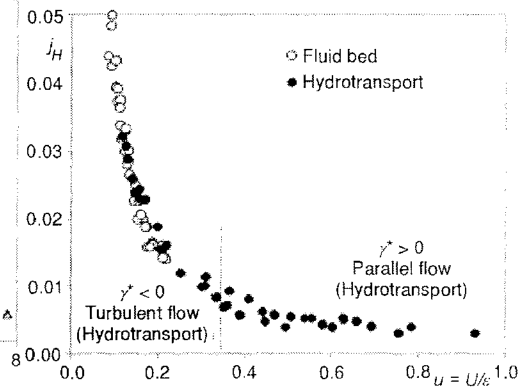

Figure 7. Relationship j_H vs. u for hydraulic transport and liquid fluidized bed

were conducted in same experimental system as shown in Fig. 1. The only difference was that transport was replaced by a fluidization column with same diameter and same heat transfer area.

Fluid-wall and particle friction

The one-dimensional mixture momentum eq. (1) outside the acceleration zone of the transport tube is

$$-\frac{dP}{dz} = (\rho_p - \rho_f) g (1-\varepsilon) + F_w \tag{11}$$

The individual momentum balances for the fluid and particle phases [15] are

$$\varepsilon \left[-\frac{dP}{dz} \right] = \beta (u - v)^2 + F_f \tag{12}$$

$$(1-\varepsilon) \left[-\frac{dP}{dz} \right] = -\beta (u - v)^2 + (\rho_p - \rho_f) g (1-\varepsilon) + F_p \tag{13}$$

where $\beta(u - v)^2$ is the hydrodynamic drag force per unit volume of suspension. F_f and F_p are pressure losses due to fluid-wall and particle-wall friction written in terms of friction factors f_f and f_p:

$$F_f = 2 f_f \rho U^2 / D_t \tag{14}$$

$$F_p = 2 f_p \, \rho_p (1-\varepsilon) v^2 / D_t \tag{15}$$

The fluid-wall friction term (F_f) was determined using eq. (14) and the Blasius friction factor correlation [16]

$$f_f = 0.0791/\mathrm{Re}^{0.25} \tag{16}$$

where the Reynolds number is based on superficial gas velocity ($\mathrm{Re} = D_t \rho_f U/\mu$). Our experiments with no particles present show excellent agreement between eq. (16) and the experimental values of f_f.

A particle-wall friction term was correlated in our non-accelerating experiments conducted in a transport tube of same diameter [7]

$$f_p = 7.33 \cdot 10^{-3} v^{-2}, \quad v \text{ in m/s} \tag{17}$$

As seen above, the individual momentum balances for the fluid and particle phases require that overall friction of the flowing mixture with the wall have the additive character

$$F_w = F_f + F_p \tag{18}$$

Figure 8 shows variation of four pressure drop ratios normalized relative to the overall pressure gradient with superficial fluid velocity, where

$$F_e = (\rho_p - \rho_f) g (1 - \varepsilon) \tag{19}$$

At low relative velocities where the solids fraction is large, the major portion of the dynamic pressure drop is due to the static head of the submerged particles (F_e). With an increase in liquid velocity, the fluid-wall friction contribution increases significantly and can be as high as about 40% of total at $U/U_t \approx 4$. For low values of U/U_t, note that F_p is greater than F_f but small relative to the overall pressure gradient. This quantity one can determined experimentally by measuring $-dP/dz$ and ε. The introduction of separate contributions F_f and F_p (with $F_w = F_f + F_p$) is essentially a convention, since only the quantity F_w can be determined experimentally.

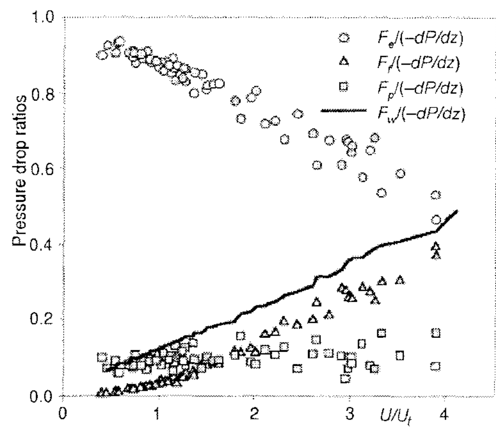

Figure 8. Variation of four pressure drop rations with superficial fluid velocity, d_p = 2.98 mm

Using experimental data for $-dP/dz$, U, and ε, collected in our previous work [7], the experimental values of F_w were determined by the difference from eq. (11)

$$F_w = -\frac{dP}{dz} - (\rho_p - \rho_f) g (1 - \varepsilon) \tag{20}$$

Treating a flowing mixture as a pseudofluid with the mean density

$$\rho_m = \varepsilon \rho_f + (1 - \varepsilon) \rho_p \tag{21}$$

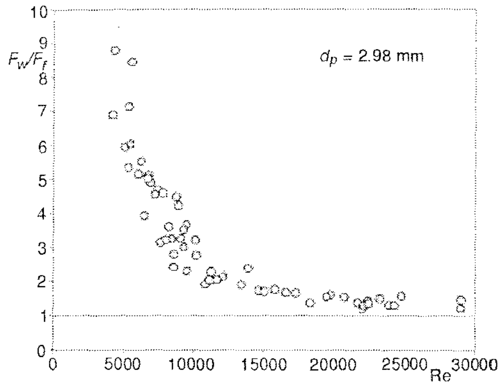

Figure 9. Variation of F_w/F_f with Reynolds number

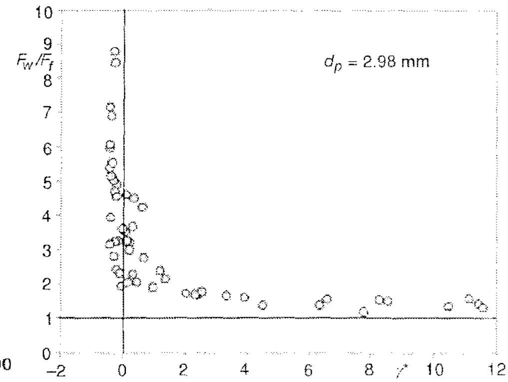

Figure 10. Variation of F_w/F_f with regime parameter γ^*

a modified mixture-wall friction coefficient can be defined by the analogy with eq. (14):

$$f_w = \frac{F_w D_t}{2\rho_m U_m^2} \tag{22}$$

where the mean mixture superficial velocity is

$$U_m = U + c_s = \frac{G_f}{\rho_f A_t} + \frac{G_p}{\rho_p A_t} \tag{23}$$

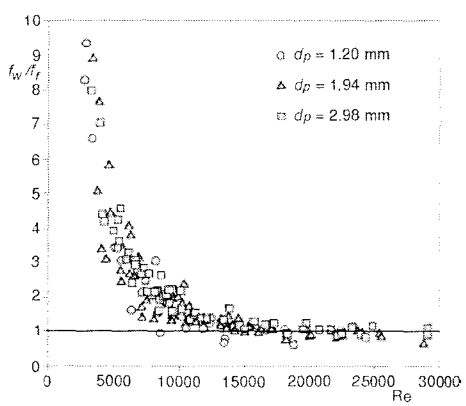

Figure 11. f_w/f_f vs. Reynolds number

Figure 9 gives variation of the ratio F_w/F_f with the tube Reynolds number. As can be seen, with an increase in Re the ratio decreases monotonically since with the increase in Re number the particle concentration in the flowing mixture also decreases. Figure 10 gives the variation of the same ratio with the regime parameter γ^*, indicating much higher friction between the flowing mixture and tube wall in the "turbulent" flow regime. As one can be expect from the above figures, the ratio of the friction coefficients f_w/f_f is a monotonically decreasing function of Reynolds number. With increase in Reynolds number, the ratio f_w/f_f asymptotically tends to 1 (Fig.11).

Correlation of data for wall-to-flowing mixture heat transfer

For the flowing mixture the modified Reynolds number is

$$\mathrm{Re}_m = \frac{D_t \rho_m U_m}{\mu_m} \tag{24}$$

where the effective flowing mixture viscosity is, according to Barnea and Mizrahi [17]

$$\mu_m = \mu \cdot \exp\left(\frac{5(1-\varepsilon)}{3\varepsilon}\right) \tag{25}$$

Figure 12 gives experimental values of j_H and experimental values of $f_w/2$ for d_p = 1.94 mm particles. As can be seen, over the range of conditions investigated these values are practically the same, clearly indicating an analogy between the two phenomena. All of the data for the j_H factor in our hydraulic transport runs are plotted against the modified mixture Reynolds number together with experimental data for the modified mixture-wall friction coefficient in Fig. 13.

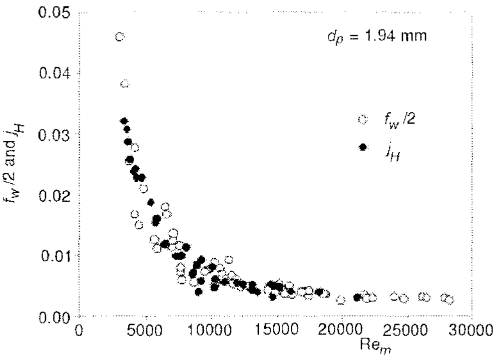

Figure 12. $f_w/2$ **and** j_H **as a function of mixture Reynolds number,** d_p **= 1.94 mm**

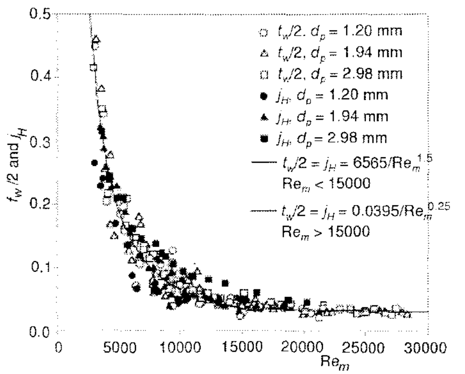

Figure 13. Correlation of the data for f_w **and** j_H **in hydraulic transport**

These data are correlated by the equations:

$$j_H = f_w/2 = 6565/\mathrm{Re}_m^{1.50}, \quad 2800 < \mathrm{Re}_m < 15000 \tag{26}$$

and

$$j_H = f_w/2 = 0.0395/\mathrm{Re}_m^{0.25}, \quad 15000 < \mathrm{Re}_m < 32000 \tag{27}$$

The form of the correlation for Re_m15000 is the same as the Blasius equation for single phase flow. The mean deviation of all data points from eq. (26) is 19.9%, and from eq. (27) 14.7%. For whole range of Re_m numbers investigated the heat transfer factors are directly proportional to mixture-wall friction coefficients, *i. e.* $j_H = f_w/2$. For single phase flow this is the case only in turbulent regime.

Conclusions

Heat transfer in hydraulic transport of coarse spherical glass particles and in single phase flow was studied. The main conclusions drawn from this investigation are as follows:

(a) In the runs with particles no present, the data for the wall-to-fluid heat transfer are correlated by the equation j_H = 3650/$Re^{1.5}$ for 2200 < Re < 10000. For the fully turbulent regime, *i. e.* for Re > 10000, the data agree well with the Chilton-Colburn's analogy $j_H = f_f/2$, where f_f is fluid wall friction coefficient, defined for smooth tubes by the Blasius equation, f_f = 0.0791/$Re^{0.25}$. A comparison of the data for single phase flow with several literature correlations shows satisfactory agreement in turbulent regime (Re > 10000). However, in transition regime (2300 < < Re < 10000) there is significant difference between available correlations and our data for heating water (D_t = 25.4 mm and D_t = 12 mm) and air (D_t = 6 mm), indicating that each correlation is system specific. These results supports view [11] that heat transfer results in the transition regime are not predictable because of the large number of parameters which determine when and how the transition occurs.

(b) The data for wall-to-bed heat transfer in hydraulic transport of coarse spherical glass particles shows that an analogy between heat and momentum transfer exists. The data were correlated treating the flowing fluid-particle mixture as a pseudofluid, by introducing modified mixture-wall friction coefficient (f_w) and modified the mixture Reynolds number (Re_m). For whole range of Re_m numbers investigated heat transfer factors are directly proportional to mixture-wall friction coefficients, *i. e.* $j_H = f_w/2$.

(c) In the hydraulic transport of coarse spherical glass particles two characteristic flow regimes were observed: "parallel" flow and "turbulent" flow regime. A parameter γ appears as the criterion for the regime distinction. In the "parallel" flow regime ($\gamma > 0$), the presence of particles has little effect on heat transfer coefficients. In "turbulent" flow regime ($\gamma < 0$), the interaction between the flowing particles and the walls seems to be more intensive and particles disturb laminar sub layer at the heating surface. As a consequence, the heat transfer coefficients significantly increase in comparison with the corresponding values for single phase flow. Experimental data show that heat transfer factor for hydraulic transport of spherical glass particles (d_p = 1.94 mm) in turbulent regime ($\gamma < 0$) and in a particulately fluidized bed of same particles are nearly the same indicating that main contribution to the heat transfer is particle –wall interaction.

Acknowledgment

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Nomenclature

A_t [m^2]	– cross-sectional area of the transport tube
C_{pf} [J/kg K]	– specific heat of fluid
C_{pp} [J/kg K]	– specific heat of solids
c_s [m/s]	– particle superficial velocity in the transport tube, $G_p/\rho_p A_t$
d_p [m]	– particle diameter
D_t [m]	– diameter of the transport tube
f	– fluid-wall friction coefficient by eq. (7), $f = 4f_f$
f_f	– fluid-wall friction coefficient
f_p	– particle-wall friction coefficient
f_w	– mixture-wall friction coefficient
F_e [Pa/m]	– pressure gradient due to the effective weight of particles
F_f [Pa/m]	– pressure gradient due to the fluid-wall friction
F_p [Pa/m]	– pressure gradient due to the particle-wall friction
F_w [Pa/m]	– pressure gradient due to the mixture-wall friction
g [m/s^2]	– gravitational acceleration
G_f [kg/s]	– fluid mass flowrate in the transport tube
G_p [kg/s]	– particle mass flowrate in the transport tube
h [kW/m^2 K]	– heat transfer coefficient,
j_D	– mass transfer factor
j_H	– heat transfer factor, $(\alpha/\rho_f C_{pf} U)\mathrm{Pr}^{2/3}$ = Nu/RePr$^{1/3}$
L [m]	– separation length between the bed bottom and transport tube inlet, *i.e.* spout height (see Fig.1)
L_H [m]	– length of heating zone
Nu	– Nusselt number, $\alpha D_t/\lambda$
P [Pa]	– dynamic pressure
Pr	– Prandtl number, $\mu C_{pf}/\lambda$
Re	– pipe Reynolds number, $D_t\rho_f U/\mu$
Re$_m$	– modified mixture Reynolds number, $D_t\rho_m U_m/\mu_m$
T [K]	– temperature
u [m/s]	– mean interstitial fluid velocity in the transport tube, U/ε
U [m/s]	– superficial fluid velocity in the spout and in the transport tube
U_m [m/s]	– superficial fluid mixture velocity, $U + c_s$
U_t [m/s]	– particle terminal velocity in an infinite medium
W_p [kg/m^{2}s]	– particle mass flux, G_p/A_t
v [m/s]	– radially averaged particle velocity in the transport tube
z [m]	– vertical coordinate

Greek letters

α [W/m^{2}K]	– heat transfer coefficient
β [kg/m^4]	– fluid-particle interphase drag coefficient
γ	– defined by eq. (7)
γ^*	– defined by eq. (8)
ε	– radially averaged voidage in the transport tube
λ [W/mK]	– water thermal conductivity
μ [Ns/m^2]	– viscosity of the fluid
μ_m [Ns/m^2]	– viscosity of the fluid-particle mixture
ρ_f [kg/m^3]	– fluid density
ρ_p [kg/m^3]	– particle density
ρ_m [kg/m^3]	– mean mixture density, $\varepsilon\rho_f + (1-\varepsilon)\rho_p$

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Authors' addresses:

R. V. Garić-Grulović, Z. Lj. Arsenijević, and S. Dj. Jovanović
Institute for Chemistry, Technology and Metallurgy,
University of Belgrade
12, Njegoševa, 11000 Belgrade,Yugoslavia

Ž. B. Grbavčić,
Faculty of Technology and Metallurgy,
University of Belgrade
4, Karnegijeva, 11000 Belgrade, Yugoslavia

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