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Theory of Infrared Detector Based on the Microcantilever Resonant Frequency Temperature Dependence

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Abstract

We developed a theory for the infrared (IR) thermal detector whose function is based on the dependence of the microcantilever resonant frequency on the thermal stress caused by IR radiation. We study performance of such a sensor using the general theory of microcantilever based resonators. The stress in the microcantilever is a result of the combined effect of the room temperature built-in stress stemming from the fabrication process and the thermal stress caused by the beam heating. We derive for the first time the analytical expressions for basic parameters, sensitivity, noise and detectivity, of such a sensor.

Keywords: IR detector; microcantilever; oscillator; detectivity; thermal stress

1. Introduction

Recently, there has been a growing interest in thermal detectors based on microcantilevers. A majority of works in this field suggest the use of bimaterial effect for detection of IR radiation [1, 2]. We present a theoretical study of an IR detector whose operation is based on the change of the resonant frequency of a doubly clamped microcantilever resonant structure caused by IR radiation, as first proposed in Ref. [3]. Methods for optimization of the thermal resistance of this type of IR detector are proposed in [4], starting from the well-known design of the bolometric MEMS IR detectors. The theory presented in this work is general, thus it is aplicable to the cases considered in Ref. [4].

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2. Theory

It is known that the resonant frequency for the first vibration mode of a rectangular doubly clamped beam under zero tension is given by $f_0 = 1.0279(t/L^2)[E/\rho]^{1/2}$, where t is the beam thickness in plane of vibration, L is the length, ρ is the density and E is the Young's modulus of elasticity of the beam material. The dimensions of the structure L, w and t are shown in Fig. 1. Under stress, the resonant frequency changes [1, 2] according to $f(\sigma) = f_0[1 + \gamma\sigma(L/t)^2/E]^{1/2}$, where σ is the tensile or compressive stress and γ is numerical coefficient ($\gamma = 0.295$). When the temperature of the beam changes, the change in the thermal stress has a significant influence on the value of the resonant frequency, due to multiplication factor $\gamma(L/t)^2/E$, which for a cantilever with $L/t = 20$ attains value of 100. The stress, σ, is the sum of the room temperature built-in stress, σ_i, resulting from the microcantilever fabrication processes, and thermal stress, σ_t, from heating of the beam by IR radiation. It is easy to show that the resonant frequency dependence on the thermal stress is given by

$$f(\sigma_t) = f_i\sqrt{1+\gamma\, f_o^2\sigma_t L^2 / f_i E t^2}\ , \tag{1}$$

where $f_i = f(\sigma_i)$. The thermal stress is $\sigma_t = \alpha E\Delta T$, where α is the thermal expansion coefficient and ΔT is the average increase in the beam temperature. Using Eq. (1) and the expression for the increase of the detector active area temperature, ΔT, in the case of harmonic excitation with frequency ω [1, 2], we derived the final expression for the resonant frequency of the cantilever exposed to IR radiation of the optical power P (ε is the active area emissivity, τ_{th} is the thermal time constant)

$$f(P) = f_i\sqrt{1+\gamma\,(f_o/f_i)^2\,\alpha\varepsilon R_{th}(L/t)^2 P \Big/ (1+\omega^2\tau_{th}^2)^{1/2}} \tag{2}$$

and then an explicit expression for the IR detector sensitivity defined as $S = \partial f(P)/\partial P$

$$S = \alpha\varepsilon R_{th}\gamma(L/t)^2 f_0^2/(2f)/\sqrt{1+\omega^2\tau_{th}^2}\ . \tag{3}$$

The performance of IR detectors is influenced by various noise generation mechanisms. Recent studies [5, 6] showed that the intrinsic cantilever noise is dominant for this kind of detectors. It comprises two independent mechanisms. The first mechanism is related to the induced stress in the cantilever due to spontaneous fluctuations of its temperature during the heat exchange with the ambient. These stress fluctuations generate resonant frequency fluctuations. The corresponding frequency noise in the bandwidth B, according to Eq. (1), is

$$\left\langle \Delta f_{nT}^2 \right\rangle = \left(f_0^2/2f_i\right)^2 \alpha^2\gamma^2 (L/t)^4 \left\langle \Delta T_n^2 \right\rangle, \tag{4}$$

where $\langle\Delta T_n^2\rangle = 4k_B T^2 R_{th} B/(1+\omega^2\tau_{th}^2)$ describes the fluctuations of the cantilever temperature [7]. The second noise mechanism is related to the cantilever stochastic thermal vibrations at the temperature T. The spectral density of the resulting frequency fluctuations are obtained by solving Langevin or corresponding Focker-Plank equation. The obtained expression for a microcantilever with quality factor Q and oscillation amplitude A_0

$$\left\langle \Delta f_{nf}^2 \right\rangle = f k_B T B / (2\pi k_{eff} Q A_0^2) \, , \tag{5}$$

completely matches the well known expression [8] for the cantilever frequency noise.

If the active area of the analyzed IR detector is A and $<\Delta f_n^2>$ is the total IR detector noise, the normalized detectivity for the field of view 2π is $D^* = (AB)^{1/2}(<\Delta f_n^2>/S^2)^{-1/2}$. Therefore, by using Eqs. (4) and (5) and by substituting the equation for the effective stiffness constant of the doubly clamped rectangular ($L \times w \times t$) beam ($k_{eff} = 16Ew(t/L)^3/(1-\nu^2)$, ν is the Poisson's ratio of the beam material) the following expression for the detectivity is obtained

$$D^* = \sqrt{A}\left[\frac{4k_B T}{\varepsilon^2 R_{th}}\left(\frac{f}{f_i}\right)^2 + \frac{f^3}{8\pi f_0^4}\frac{k_B T}{Q}\frac{1}{\gamma^2}\left(\frac{t}{L}\right)\frac{(1-\nu^2)}{E}\frac{(1+\omega^2\tau_{th}^2)}{\varepsilon^2 R_{th}^2 w \alpha^2 A_0^2}\right]^{-\frac{1}{2}} . \tag{6}$$

3. Numerical calculations

We apply the presented theory on an IR thermal detector based on a microcantilever resonant structure, shown in Fig. 1. The detector's performance has been studied for the structures fabricated of different materials (Al, Ni or Si). Thermophysical properties of the chosen materials are listed in Table 1.

Table 1. Thermophysical properties of Al, Ni and Si.

Material	Young's modulus $E(x10^9 Pa)$	Expansion coefficient $\alpha(x10^{-6}\,K^{-1})$	Thermal conductivity $g(xWm^{-1}K^{-1})$	Heat capacity $c(x10^6\,Jm^{-3}K^{-1})$	Density $\rho(x10^{-3}\,kg\,m^{-3})$
Al	70	25	237	2.43	2.7
Ni	200	13	90	4.8	8.9
Si	170	2.6	150	1.65	2.4

Using the derived analytical expressions, normalized sensitivity, S/ε, and normalized detectivity, D^*/ε, were calculated for the detector with the active area $A = 50 \times 30$ μm^2 (the cantilever dimensions $50 \times 30 \times 1.5$ μm^3) and for the cantilever oscillation amplitude $A_0 = 100$ nm, at ambient temperature $T = 300$ K. Of practical interest is detection of low incident optical power for which one can make a justified assumption that $f \approx f_i$ (Eq. (2)). For the analyzed detector with sensing element made of Al, whose thermal resistivity is $R_{th} = 5 \times 10^4$ K/W when all of the thermal loss mechanisms are present, the calculated normalized sensitivity is 1.33×10^9 Hz/W, assuming Q = 100. For the detector design optimized in the sense of minimization of thermal losses [4], R_{th} has a maximal value of 10^8 K/W, resulting in sensitivity which is three orders of magnitude higher.

Fig. 2 shows both dependences of the normalized sensitivity and the normalized detectivity on thermal resistivity, for the three different materials from which microcantilever is made of.

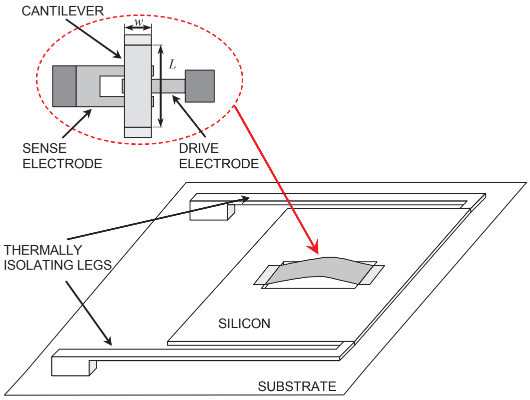

Fig. 1. Schematic of the structure of a single microcantilever resonant IR detector. The design is convenient for formation of the detector matrix.

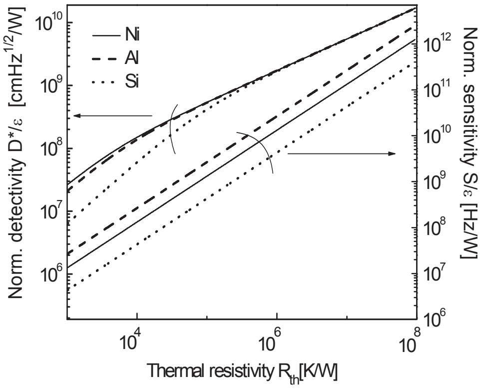

Fig. 2. Normalized detectivity, D^*/ε, for $\omega<<1/\tau_{th}$ (left axis) and normalized sensitivity, S/ε, (right axis) as a function of thermal resistivity R_{th} for three different materials (Al, Ni, Si).

It can be seen that the detectivity improves with increasing thermal resistivity. After reaching a specific value of R_{th} detectivity virtually varies as $R_{th}^{1/2}$ (the first term in Eq. (6) dominates). This limiting value of R_{th} decreases, as is it can be seen from Eq. (6), with increasing Q factor. The Q factor can be improved by evacuation of the detector package [2].

4. Conclusion

The derived theory of IR thermal detector based on the doubly clamped beam, whose resonant frequency changes due to thermal stress when exposed to IR radiation, is presented for the first time. The theory enables determination of the sensitivity and detectivity of this type of thermal detector and analysis of their dependence on various parameters (related to the detector design and operating conditions), in order to achieve optimization of the detector performance.

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