# Joint Effect of Heterogeneous Intrinsic Noise Sources on Instability of MEMS Resonators

Olga Jakšić, Ivana Jokić, Miloš Frantlović, Danijela Randjelović, Dragan Tanasković, Žarko Lazić and Dana Vasiljević-Radović

***Abstract*—This article's focus is on the numerical estimation of the overall instability of microelectromechanical-system-based (MEMS) resonators, caused by intrinsic noise mechanisms that are different in nature (electrical, mechanical or chemical). Heterogeneous intrinsic noise sources in MEMS resonators that have been addressed here are Johnson–Nyquist noise, 1/f noise, noise caused by temperature fluctuations and adsorption-desorption induced noise. Their models are given first (based on analytical modeling or based on empirical expressions with experimentally obtained parameters). Then it is shown how each one contributes to the phase noise, a unique figure of merit of resonators instability. Material dependent constants $\alpha$ and knee position in noise spectrum, needed for empirical formulae referring to 1/f noise, have been obtained experimentally, by measurements of noise of MEMS components produced in the Centre of Microelectronic Technologies of the Institute of Chemistry, Technology and Metallurgy in Belgrade. According to these measurements, $\alpha$ varies in the range from $0.776 \cdot 10^{-4}$ to $2.26 \cdot 10^{-4}$ and cut off frequency for 1/f noise varies from 147 Hz to 1 kHz. The determined values are then used for the modeling of micro-resonator phase noise with electrical origin and overall phase noise of a micro-resonator. Numerical example for calculation of overall phase noise is given for a micro-cantilever, produced by the same technology as measured components. The outlined noise analysis can be easily extended and applied to noise analysis of MEMS resonator of an arbitrary shape.**

***Index Terms*—1/f noise, adsorption, desorption, temperature fluctuations, intrinsic noise, Johnson noise, micro cantilever, Nyquist noise, phase noise, power spectral density, thermal noise, thermo-mechanical noise, white noise.**

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## I. Introduction

THE importance of “micro-electro-mechanical-systems” i.e. MEMS and “micro-opto-electro-mechanical-systems” i.e. MOEMS devices grows over time in many fields. Miniaturization of components, enabled by nanotechnologies, ensured further integration of oscillators with ever smaller passive components and other functional blocks. As a consequence, we have lower power consumption but also tunability, new functionalities [1] and better overall performances of many devices: new nano-resonators can have their fundamental resonance frequency of over 10 MHz [2] and a quality factor reaching $2 \cdot 10^4$ [3], new ultrathin silicon cantilevers can detect attonewton forces [4] and new sensors ensure mass detection of a single molecule [5]. Limiting performances of these devices are governed by intrinsic noise sources [6] (assuming extrinsic noise sources are suppressed by shielding and other noise reduction techniques [7]).

The mechanisms of intrinsic noise in MEMS and NEMS (“nano-electro-mechanical-systems”) devices are diverse. They differ in physical nature, in magnitude, in mathematical approach suitable for their modeling, they behave differently with scaling and the balance between extrinsic and intrinsic noise becomes also different with scaling (in an integrated circuit all noise mechanisms of its constituent parts are intrinsic). They all contribute to the same output signal, and it is important to know how the joint effect of all of them determines limiting performances and also how it affects device's optimization and characterization.

The often met constituent parts in MEMS and NEMS devices are resonant structures: cantilevers, beams, membranes and “comb-like” structures. In this work we consider the cantilever as the basic part.

This paper focuses on the theoretical modeling of intrinsic noise sources of a micro-cantilever with the piezo-resistive read-out and presents experimental determination of technology-dependent parameters appearing in theoretical models. The second section addresses fluctuations in different domains: the electrical noise induced by fluctuations in movement of free carriers, Johnson-Nyquist and 1/f noise, noise caused by temperature fluctuations, noise induced by fluctuations in mechanical displacement of moving parts in a MEMS resonator and noise induced by resonator's mass fluctuations due to adsorption-desorption processes. This is followed by a section addressing the phase noise, a unique figure of merit for the estimation of the resonator instability where the connection of heterogeneous intrinsic noise mechanisms with phase noise is given. The paper finally considers the application of the outlined theory.

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## II. Theory

### A. Thermal noise

Thermal noise is an intrinsic noise mechanism that exists in all MEMS devices with electrical parts, or to be more precise in all MEMS devices with free carriers and an electrical read-out. Even if there were no current flowing through the device, if there is an electrical read-out, the output voltage would be the consequence of the thermal motion of free carriers.

Thermal noise was first studied experimentally in 1928 by John B. Johnson [8] at Bell Telephone Laboratories. Analytical explanations of Johnson's experiments from a thermodynamicall point of view were given by his colleague Harry Nyquist [9], also at Bell Telephone Laboratories. Thus thermal noise is often called Johnson noise, Nyquist noise or Johnson-Nyquist noise.

According to Nyquist theory, the power spectral density of voltage fluctuations ($S_{VJ}$) stems from thermal agitation of electric charges in MEMS resistors whose resistance is $R$, at temperature $T$ and frequency $f$. It is proportional to the quadruple product of that resistance and the total energy per each degree of freedom:

$$S_{VJ} = 4R \frac{hf}{e^{hf/k_B T} - 1} \qquad \left( V^2/Hz \right). \tag{1}$$

where $h$ is Plank constant and $k_B$ is Boltzmann constant.

Based on the fluctuation-dissipation theorem developed by Callen and Welton [10] in 1951, the Nyquist theorem may be generalized. An electrical circuit is a linear system and the same formula is valid for an arbitrary electrical circuit with an impedance $Z$, and in that case the real part of $Z$ is put in the expression (1) instead of $R$. In practical applications where the total energy per degree of freedom may be considered constant (equal to $k_BT$) within specified ranges of frequency and temperature, the following formula is valid:

$$S_{VJ} = 4k_B T \,\mathrm{Re}\{Z\} \qquad \left( V^2/Hz \right). \tag{2}$$

In case of a pure resistor, this equation leads to a flat power spectrum, so thermal noise is also known as a white noise.

### B. 1/f Noise

Prior to his famous work on experimental observations of thermal noise published in *Physics Review* in 1928 [8], Johnson wrote a letter to *Nature* [11] where he commented on voltage fluctuations that “appears to be the result of thermal agitation of the electric charges in the material of the conductor”. In that letter, he also commented on fluctuations that depend not on temperature but inversely on frequency — so we may say today that Johnson had discovered electrical '1/$f$ noise' known also as flicker or pink noise. Flicker noise has remained the topic of active theoretical and experimental research till today [12]. The overall agreement is that, contrary to thermal noise, flicker noise is a non-equilibrium phenomenon, it appears in the presence of polarization in electrical circuits. Its probable cause is assumed to be scattering of free carriers during the transport through conducting material [13]. In this paper, we describe the power spectral density of voltage fluctuations induced by flicker noise by the empirical expression introduced by F. N. Hooge [14]:

$$S_{VH} = \frac{\alpha V_B^2}{Nf} \qquad \left( V^2/Hz \right) \tag{3}$$

where $V_B$ stands for the bias of the electrical circuit, $f$ is frequency, $N$ is the number of free carriers and alpha is a material-dependent constant that has to be determined experimentally. According to [15], alpha ranges from $10^{-3}$ to $10^{-7}$, depending on the material.

### C. Temperature Fluctuations-Induced Noise

Noise induced by temperature fluctuations is another example of how the fluctuation-dissipation theorem can be applied to any linear dissipative system in thermodynamic equilibrium. According to derivations outlined in [16], [17], the expression for the power spectral density of temperature fluctuations is:

$$S_{\Delta T} = \frac{4k_B T^2 G_{th}}{\left( G_{th}^2 + 4\pi^2 f^2 C_{th}^2 \right)} \qquad \left( K^2/Hz \right). \tag{4}$$

Here $C_{th}$ is the heat capacity calculated as a product of the specific heat and the volume of the observed structure and $G_{th}$ stands for thermal conductivity which is calculated using the Steffan-Boltzmann law. The power spectral density of this noise is Lorenzian, its typical shape being shown in Fig. 1.

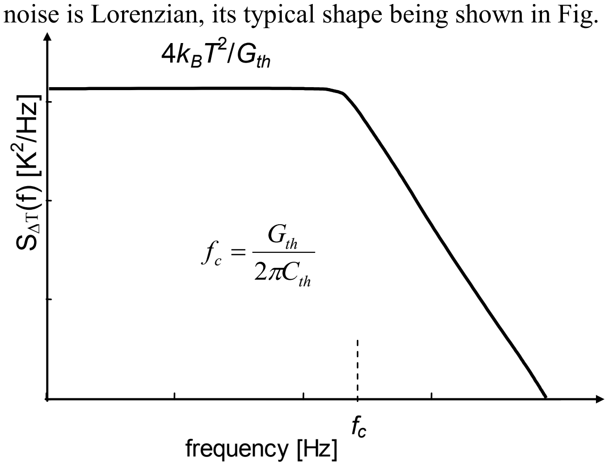


Fig. 1. Power spectral density of temperature fluctuations in a microstructure with a thermal conductivity $G_{th}$ and a heat capacity $C_{th}$.

### D. Thermo-mechanical Noise

Thermo-mechanical noise is a consequence of mechanisms of energy transfer between the structure of a mechanical resonator and its surroundings in equilibrium. The detailed derivation outlined in [18] and in literature stated therein leads to the following expression for the power spectral density of the displacement fluctuations of a micro-resonator:

$$S_X = \frac{4k_B T k}{Q\omega_0} \frac{1/m_r^2}{\left( \omega_0^2 - 4\pi^2 f^2 \right)^2 + \left( \frac{\omega_0 2\pi f}{Q} \right)^2} \qquad \left( m^2/Hz \right). \tag{5}$$

$Q$ is quality factor, $m_r$ is the resonators mass, $k$ is the resonator stiffness constant, and $\omega_0$ the resonant angular frequency. The meaning of other terms is the same as above. A typical spectrum is shown in Fig. 2.

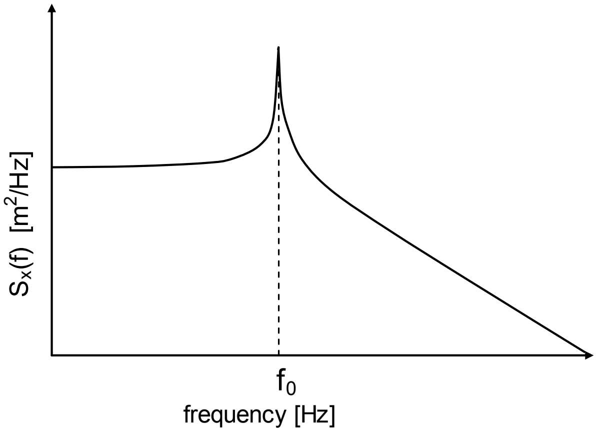


Fig. 2. Power spectral density of displacement fluctuations in a micro-resonator with a resonant frequency $f_0$.

### E. Adsorption-desorption Induced Noise

The last important intrinsic noise source considered here is the adsorption-desorption noise. It is a mass fluctuation-induced noise and its importance grows as the resonator dimensions scale down so it may, under some conditions, be the dominant noise mechanism in a micro-resonator. It has been first observed by Yong and Vig [19]. In 2002, based on an analogy between adsorption-desorption processes of molecules on a solid surface and generation-recombination processes of charge carriers in a semiconductor, Djurić et al developed the first mathematical model of the power spectral density of mass fluctuations caused by monolayer mono-component adsorption of surrounding gas molecules on the resonator surface [20]:

$$S_{\Delta m} = \frac{4M_a^2 C_2 N_m bpA}{C_2^2 (bp+1)^3 + 4\pi^2 f^2 (bp+1)} \quad \left( {kg^2}/{Hz} \right). \tag{6}$$

Here $M_a$ is the mass of a gas molecule, $C_2$ is the constant inversely proportional to the mean residential time of adsorbed molecules on the surface, $N_m$ is the maximal number of adsorption sites on the surface, $A$ is the resonator surface area, $p$ is pressure and $b$ is calculated as

$$b = \frac{\alpha_s \tau}{N_m} \frac{3.513 \cdot 10^{22}}{\sqrt{M_a T}} \tag{7}$$

Expression (7) is valid if pressure is given in Torr, and $\alpha_s$ here is the sticking coefficient representing the affinity of gas molecules towards the surface and $\tau$ is the mean residential time of adsorbed gas molecules on the resonator surface.

Typical spectrum for power spectral density of resonator mass fluctuations caused by adsorption-desorption noise has a Lorenzian shape. It is shown in Fig 3.

The model of bi-component monolayer adsorption-induced micro-cantilever mass fluctuations is developed in [21], and Fig 4 shows the typical spectrum where one can distinguish two knees suitable for noise spectroscopy.

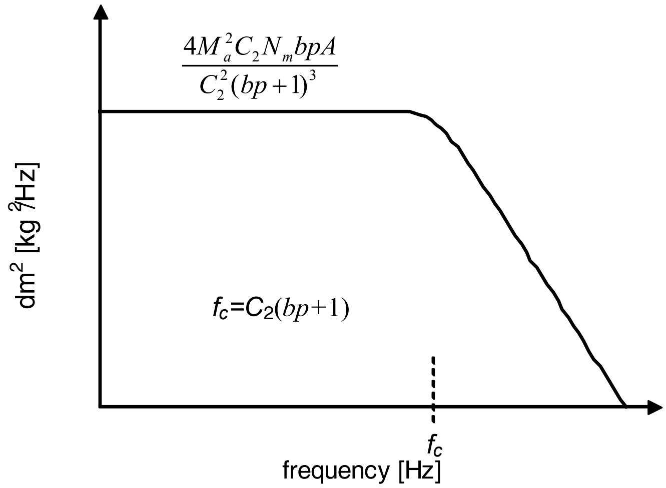


Fig. 3. Power spectral density of resonator mass fluctuations caused by nanocomponent monolayer adsorption of surrounding gas on resonator surface.

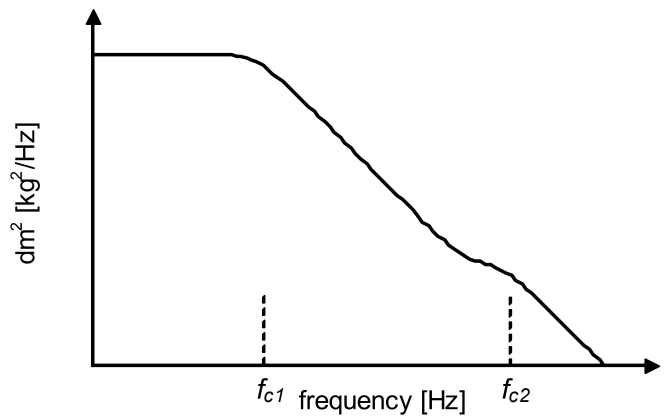


Fig. 4. Power spectral density of resonator mass fluctuations caused by bicomponent monolayer adsorption of surrounding gas on resonator surface.

## III. Phase Noise

The figure of merit for the quality of a micro-resonator is its phase noise. The definition, based on the double-sideband power spectral density of phase fluctuations $S_\phi$, is given by the standard IEEE 1139 (IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities) [22]:

$$\mathcal{L}(f) = \frac{1}{2} S_\phi(f) \tag{8}$$

The overall power spectral density of phase fluctuations of a micro-resonator is calculated as a sum of all power spectral densities of phase fluctuations calculated for every particular intrinsic noise mechanism separately. After summing, the result is often given in decibels:

$$\mathcal{L}(f) = 10 \log\left( \frac{1}{2} S_\phi(f) \right) \tag{9}$$

Here $f$ is, just as in theory of phase noise in electrical oscillators [23], the distance (along the frequency axis) from the resonant frequency, as shown in Fig. 5.

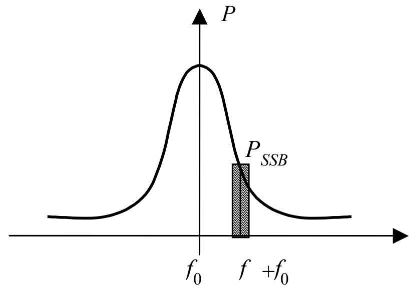


Fig. 5. Power spectral density of an electrical oscillator. Phase noise is defined as the ratio of $P_{SSB}$ (the single-sided power emitted at the frequency $f$ from a resonant frequency $f_0$ in a bandwidth of 1 Hz) and $P$ (the overall power emitted when the oscillator is in its resonant mode).

Only after establishing the connection of heterogeneous fluctuations caused by different mechanisms of intrinsic noise with the power spectral density of phase fluctuations it is possible to find their joint effect on micro-resonator stability.

### *A. Phase Noise Induced by Voltage Fluctuations*

Voltage fluctuations in micro-mechanical resonators with electrical read-out may be treated according to the theory of electrical oscillators. The modeling of connection between voltage fluctuations due to thermal and flicker noise and phase fluctuations in electrical oscillators is empirically established by Leeson [24]. According to Leeson, phase noise in oscillators has two origins: up-conversion of electrical noise (flicker and Johnsons) to the high-frequency region around the resonant frequency and phase fluctuations that modulate the resonant frequency. Their effect on phase fluctuations of an oscillator depends on the resonant frequency $f_0$ and the quality factor $Q_L$ (index $L$ refers to a closed loop):

$$S_{\phi Osc}(f)=S_{\phi Amp}\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right)=\frac{S_V(f)}{P_0}\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right)$$
$$=\frac{S_{VJ}}{P_0}\left(1+\frac{f_c}{f}\right)\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right) \tag{10}$$

An oscillator is attained as an amplifier in a frequency selective closed loop and the connection between phase fluctuations of an amplifier $S_{\phi Amp}$ and voltage fluctuations $S_V$ is $P_0$, the power of the signal at the resonant frequency.

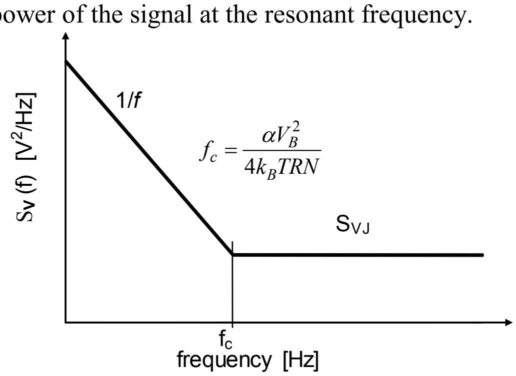


Fig. 6. Thermal and flicker noise before up-conversion to region around resonant frequency

In a micromechanical resonator voltage fluctuations have the same form: spectrum produced by the calculation of thermal and 1/f noise using (1) or (2) and (3) is given in Fig. 6 and has the same form of a spectrum of an amplifier in an open loop $S_{\phi Amp}$ used in (10). The connection between the phase fluctuations and the phase fluctuations in a mechanical microresonator is done by taking into account the displacement fluctuations.

Here we consider the piezoresistive read-out made by a Wheatstone bridge where one of four identical resistors is placed on the moving resonating part of a cantilever and all three other resistors are placed on the steady base of a cantilever. The output voltage equals one quarter of the product of the bias voltage $V_B$, displacement $X$ and cantilevers voltage sensitivity to displacement $S$. In [26] Cleland and Roukes showed that the power spectral density of phase fluctuations equals the power spectral density of the displacement fluctuations divided by the mean square amplitude of oscillations $\left\langle X_0^2\right\rangle$.

Phase noise in microresonator due to thermal and flicker noise is then

$$S_{\phi\{V\}}(f)=S_\phi\left(S_{\Delta X}\left(S_V\right)\right)\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right)$$
$$=S_\phi\left(\frac{16S_V}{S^2V_B^2}\right)\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right)=\frac{16S_V}{\left\langle X_0^2\right\rangle S^2V_B^2}\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right) \tag{11}$$
$$=\frac{16S_{VJ}}{\left\langle X_0^2\right\rangle S^2V_B^2}\left(1+\frac{f_c}{f}\right)\left(1+\left(\frac{f_0}{2fQ_L}\right)^2\right)$$

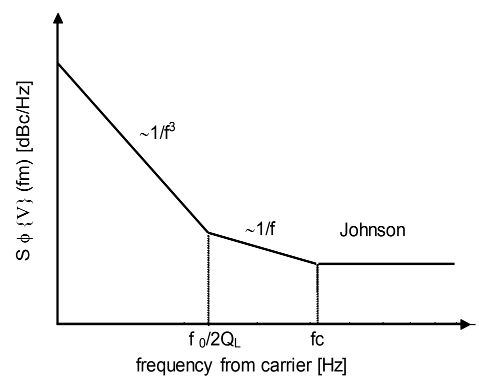


Fig. 7. Power spectral density of phase fluctuations for an oscillator with a high quality factor

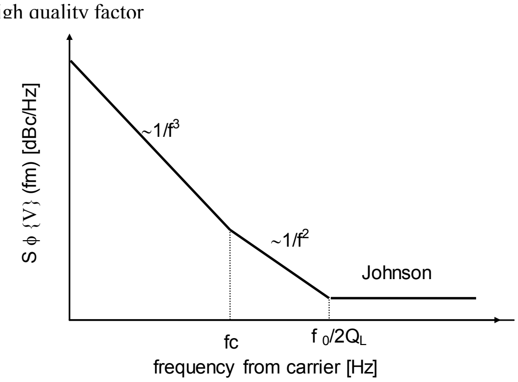


Fig. 8. Power spectral density of phase fluctuations for an oscillator with a low quality factor

In general the spectrum has a shape like that in Fig. 7 or Fig 8.depending on the values for $f_c$ and $f_0/(2Q_L)$.

Oscillators with a low quality factor have the diagram as shown in Fig. 8 where the first knee corresponds to $f_c$ and the second to $f_0/(2Q_L)$. The knee positions are reversed for oscillators with high quality factors. In both cases, in order to apply (11) the knee position must be determined experimentally or estimated based on good knowledge of material properties.

### *B. Phase Noise Induced by Temperature Fluctuations*

According to reasoning by Vig in [25], fluctuations of the normalized frequency of the resonator are proportional to fluctuations in temperature with a factor of proportionality $\alpha_T$, so the power spectral densities of these fluctuations relate as:

$$S_{y\{\Delta T\}}(f) = \alpha_T^2 S_{\Delta T}(f) \tag{12}$$

The power spectral density of resonator phase fluctuations induced by fluctuations in temperature is then

$$S_{\phi\{\Delta T\}}(f) = \frac{f_0^2}{f^2} S_{y\{\Delta T\}}(f) = \frac{f_0^2}{f^2} \alpha_T^2 S_{\Delta T}(f) \tag{13}$$

Notation is as stated before.

### *C. Phase Noise Induced by Displacement Fluctuations*

Let us observe the power spectral density of displacement fluctuations given by expression (5) and shown in Fig. 2 in the vicinity of the resonant frequency:

$$S_X(f+f_0) = \frac{2k_B T}{Qk\pi} \frac{f_0^3}{\left(f^2+2f_0 f\right)^2 + \left(\frac{f_0^2}{Q}+\frac{f_0 f}{Q}\right)^2}. \tag{14}$$

The power spectral density of phase fluctuations is the ratio of the power spectral density of displacement fluctuations and the mean square of the amplitude $X_0$ of mechanical oscillations [26]:

$$S_{\phi\{X\}}(f) = \frac{S_X(f+f_0)}{\left\langle X_0^2 \right\rangle}. \tag{15}$$

In the frequency range of interest it is valid that

$$\frac{f_0}{Q} \; \square \quad \square \quad . \tag{16}$$

Bearing that in mind the derivations outlined in [18], [25], [26] and in references therein, one comes to the final expression for the power spectral density of phase fluctuations caused by fluctuations in displacement of a mechanical resonator.

$$S_{\phi\{X\}}(f) = \frac{k_B T f_0^2}{Q^2 P_C f^2}. \tag{17}$$

Here $P_C$ is the resonators power dissipation calculated using the Stefan–Boltzmann law.

### *D. Phase Noise Induced by Mass Fluctuations*

Mass fluctuations directly affect the resonant frequency [20]:

$$f_0 = \sqrt{\frac{k}{m_0+\Delta m}} \cong f_0\left(1-\frac{\Delta m}{2m_0}\right). \tag{18}$$

$m_0$ is the overall mass of the resonator and $\Delta m$ is the adsorption-induced mass change. The corresponding power spectral density of phase fluctuations of a micro-resonator is then [27]:

$$S_{\phi\{\Delta m\}}(f) = \frac{f_0^2}{f^2} S_y(f) = \frac{f_0^2}{f^2}\frac{S_{\Delta f}(f)}{f_0^2} = \frac{1}{f^2}\frac{S_{\Delta m}(f) f_0^2}{m_0^2}. \tag{19}$$

Now (6) can be used in case when single-component monolayer adsorption takes place on the surface of a micro-resonator. Alternatively, expressions developed in [21] can be used if bi-component monolayer adsorption occurs. One could also use another appropriate expression for the power spectral density of resonator mass fluctuations caused by various adsorption-desorption processes (arbitrary mixture of gases [28], multilayer adsorption [29], adsorption–desorption processes coupled with mass transfer and surface diffusion [30], etc.).

## IV. Experiments and Results

Electrical noise measurements of SP4, SP6 and SOI-SP9 and SP9 pressure sensors produced in ICTM-CMT, Belgrade, Serbia and of pressure sensors produced by KELLER, Switzerland were performed in order to characterize those already produced devices, but also in order to estimate the flicker noise knee and factor alpha needed for the theoretical investigation and the design of micro-cantilevers that may be produced by using the same technology.

The experimental set-up for these electrical noise measurements is shown in Fig. 9

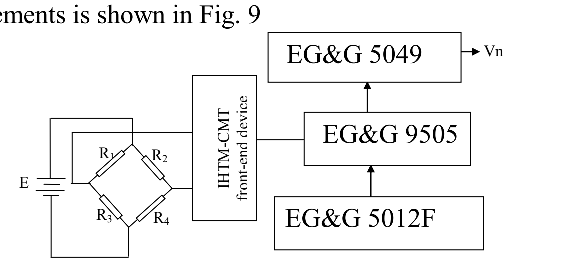


Fig. 9. Experimental set-up for electrical noise measurements. EG&G 5049 is a noise measurement unit, EG&G 9505 is a lock-in analyser and EG&G 5012F is a reference oscillator unit

The Wheatstone bridge is composed of four p-type piezo resistors formed by boron diffusion in n-Si substrate. The piezo resitors are placed on membrane fabricated using bulk micromachining. Voltage supply $E$ has been built out of 1.5V batteries in order to avoid power line interference, hence its values were 1.5V, 3V, 4.5V, 6V, 7.5V, 9V, 10.5V and 12V - Table I. An analog front-end, designed in ICTM-CMT has been used for signal interface from a sensor under test to a lock-in analyser EG&G 9505. A reference oscillator for lock-in analyser was EG&G 5012F. Measured noise voltage was recorded from a lock/ins noise measurement unit EG&G 5049.

For every measured sensor, each obtained noise spectrum has been fitted with $y = P1 + P2/x$ . P1 corresponds to thermal noise and P2 corresponds to flicker noise. Fig. 10 shows a typical noise spectrum.

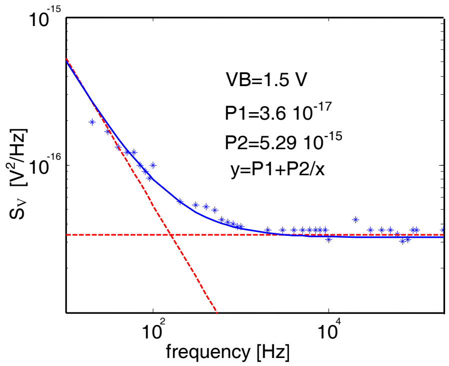


Fig. 10. Symbols: power spectral density of measured voltage fluctuations for SP4 ICTM-CMT pressure sensor, lines: fitting results, voltage supply is 1.5 V

Table I shows some of the results obtained for flicker noise parameter alpha in Hooge's formula, expression (3).

TABLE I
ALPHA PARAMETER IN HOOGE'S FORMULA FOR FLICKER NOISE

| $V_B$(V) | P1($10^{-17}$) | P2($10^{-15}$) | $\alpha$ | $f_c$ (Hz) |
|---|---|---|---|---|
| 1.5 | 3.6 | 5.29 | 2.26 | 146.944 |
| 3 | 4.9 | 7.26 | 0.776 | 354 |
| 4.5 | 7.22 | 30.25 | 1.436 | 418.7 |
| 6 | 8.1 | 49 | 1.3 | 605 |
| 7.5 | 11.025 | 88.36 | 1.51 | 801.5 |
| 9 | 12.1 | 100 | 1.19 | 826 |
| 10.5 | 16.9 | 169 | 1.47 | 1000 |

Considering all measurements of CMT components based on boron-doped n-type silicon wafers, the obtained numerical values for the alpha parameter were in a range from $0.776 \cdot 10^{-4}$ to $3.8514 \cdot 10^{-4}$. The results for alpha obtained after measurements of components produced by KELLER were in a range from $1.899 \cdot 10^{-5}$ to $1.4603 \cdot 10^{-4}$.

Obtained values were used for modeling and investigations of limiting performances of a piezo resistive microcantilever shown in Fig. 11. Two cantilevers were considered. The first one, PMG1, has the first mode resonant frequency of 62 kHz and size of 200 µm x 2 µm x 50 µm. The second one, PMG2, has the first mode resonant frequency of 110 kHz and a size of 150 µm x 2 µm x 50 µm.

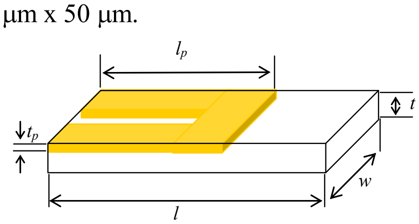


Fig. 11. Schematics of a piezo resistive micro-cantilever

It is assumed that in working conditions microcantilevers are surrounded by a mixture of gases. Table II shows the parameters of gases used in expressions related to adsorption-desorption process. The mean residential time of adsorbed molecules, needed for (7), is calculated using the expression:

$$\tau = \tau_0 e^{E_d / RT} . \tag{20}$$

TABLE II
GAS PARAMETERS

| Symbol | Quantity | Meaning |
|---|---|---|
| $M_{a1}$ | $46.48 \cdot 10^{-27}$ kg | mass of a single molecule of nitrogen |
| $M_{a2}$ | $3.32 \cdot 10^{-27}$ kg | mass of a single molecule of hydrogen |
| $N_{m1}$ | $8 \cdot 10^{18}$ cm$^{-2}$ | surface density of adsorbed $N_2$ molecules |
| $N_{m2}$ | $20 \cdot 10^{18}$ cm$^{-2}$ | surface density of adsorbed $H_2$molecules |
| $\alpha_s$ | 1 | sticking coefficient of gas molecules to surface |
| $R$ | 1.978 cal/Kmol | universal gas constant |
| $E_{d1}$ | 15.5 kcal/mol | desorption energy of nitrogen molecules |
| $E_{d2}$ | 8 kcal/mol | desorption energy of hydrogen molecules |
| $\tau_0$ | $10^{-13}$ s | period of crystal lattice vibrations |

Fig. 12 shows typical results for phase noise components calculated in order to estimate the level of instability of a micro-cantilever with piezo resistive read-out. The results are given for the piezo-resistive micro-cantilever PMG1. *E* stands for the intrinsic noise sources induced by circuits for electrical read-out, i.e. Johnson-Nyquist and flicker noise, calculated using the experimentally obtained alpha parameter in Hooge's formula and Leeson's model with the quality factor of a resonator in a closed loop $Q$ = 100. T stands for phase noise induced by temperature fluctuations. TM stands for thermo-mechanical noise and AD is for noise induced by adsorption and desorption of nitrogen molecules at a pressure of 10 Pa and hydrogen molecules at a pressure of 0.001 Pa.

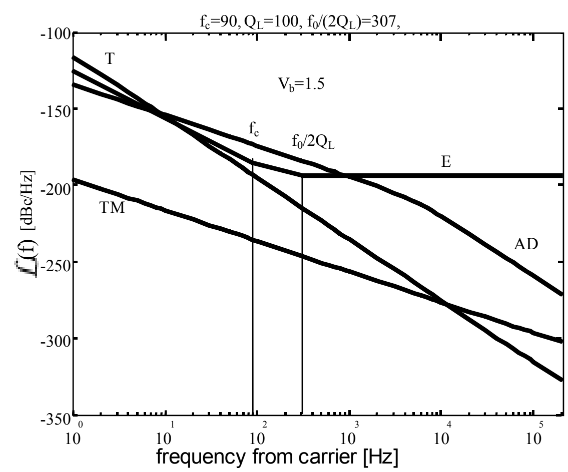


Fig. 12. Phase noise of a micro-cantilever induced by: thermomechanical noise (TM), fluctuations in temperature (T), voltage fluctuations due to Johnson-Nyquist and flicker noise (E) and adsorption-desorption noise (AD) due to traces of nitrogen (at 10 Pa) and hydrogen (0.01 Pa) in the vicinity of the piezo resistive micro-cantilever surface

## V. DISCUSSION AND CONCLUSIONS

Noise in MEMS and NEMS has been studied in 1999 [33] and after a decade it is stil a topic of great importance for many applications [34]. Experimental and numerical results given in this paper demonstrate an application of a systematic theoretical apparatus for the complete analysis of heterogeneous intrinsic noise in an arbitrary micro-resonator and may be also applied to an arbitrary micro-electro-mechanical structure including similar resonating parts: micro-cantilevers, micro-bridges, micro-membranes etc.

The outlined theory enables a comparative analysis of heterogeneous intrinsic noise mechanisms in micro-electro-mechanical resonators. The results imply that with scaling, in MEMS components, some intrinsic noise mechanisms may be promoted. This is in particular valid for adsorption-desorption noise that is insignificant in larger devices, but in certain frequency ranges in MEMS devices may prove itself dominant.

It has been shown how one can estimate the joint effect of separate noise sources on limiting performances of MEMS resonators.

The experimentally obtained values of the parameter alpha in Hooge's formula for power spectral density of flicker noise fit well in ranges for alpha reported in literature: in [14] alpha is $2\cdot10^{-3}$, in [31] alpha is ranged from $3\cdot10^{-6}$ to $3\cdot10^{-4}$ and in [32] alpha ranges from $10^{-7}$ to $10^{-3}$ depending on the material (polycristalline, crystalline, amorphous silicon).

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